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CONTENTS

CHAPTER I

STRESSES AND STRAINS

PAGE

Meaning of Stress and Strain—Measurement of Simple Tensile and Compressive Stresses—Ultimate or Breaking Stress—Factor of Safety—Dead and Live Loads—Elasticity—Measurement of Simple Tensile and Compressive Strains—Hooke's Law—Young's Modulus of Elasticity—Fatigue—Tensile Test of a Piece of Iron or Steel—Apparent and Actual Stress 1

CHAPTER II

PROPERTIES OF IMPORTANT ENGINEERING MATERIALS

Tenacity, Ductility, etc.—Cast Iron—Wrought Iron—Steel—Alloy Steels—Special Steels—Copper and other Engineering Materials . . . 16

CHAPTER III

CALCULATION OF SIMPLE STRESSES

Calculating Stresses and Strains—Worked Examples—Calculations of Moduli of Elasticity 36

CHAPTER IV

FURTHER CONSIDERATION OF STRESS AND STRAIN—COMPOUND AND REPEATED STRESSES

Normal and Tangential Stresses—Shear Stresses—Complex Stress—Ellipse of Stress—Shear Stress combined with Tensile or Compressive Stress—Different Forms of Strain—Poisson's Ratio—Stress in Compound Bars—Resilience—Stresses caused by Live and Impulsive Loads—Repeated Stresses—Wöhler's Experiments—Stresses due to the forces of Expansion and Contraction 45

Contents

CHAPTER V

STRENGTH OF CYLINDRICAL VESSELS EXPOSED TO FLUID PRESSURE

PAGE

Bursting Forces—Strength of thin Cylindrical Shell—Strength of thin Spherical Shell—Thin Oval Shell—Cylindrical Vessels exposed to external Fluid Pressure—Strength of Flat Plates 83

CHAPTER VI

RIVETED JOINTS

Strength of Single-Riveted Lap Joints—Double-Riveted Lap Joints—Treble-Riveted Lap Joints—Single-Riveted Butt Joints—Double-Riveted Butt Joints—Treble-Riveted Butt Joints—Riveted Joints for Girder Work—Board of Trade Requirements concerning Marine Boilers 97

CHAPTER VII

SIMPLE MACHINE DESIGNS

SUSPENSION LINKS, COTTERED JOINTS, AND FOUNDATION BOLTS

Strength of Suspension Links—Cottered Joints—Foundation Bolts . . . 154

CHAPTER VIII

STRENGTH OF SHAFTS

Twisting Moment—Strength of Shafts—Moment of Inertia—Power transmitted by Shafts—Comparison of Solid and Hollow Shafts—Stiffness of Shafts—Board of Trade Rules for Shafts—Shafts subjected to both Bending and Twisting—Strength of Shaft Coupling Bolts 172

CHAPTER IX

Shock or Impact Tests on Iron and Steel 211

Contents

CHAPTER X

CHAINS AND ROPES

PAGE

Fatigue of Chains—Proof Stress—Wire Ropes—Horizontally Suspended Chains and Ropes—Length of Suspended Chain or Rope—Worked Examples . 215

CHAPTER XI

REVOLVING RING

Revolving Ring—Worked Examples 227

CHAPTER XII

SQUARE SHAFTS

Square Shafts—Worked Examples 232

CHAPTER XIII

SPRINGS

Helical Springs—Worked Examples 237

CHAPTER XIV

STRENGTH OF THICK CYLINDERS

Strength of Thick Cylinders—Worked Examples 245

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Chapter I

STRESSES AND STRAINS

1.—Whenever a force or a number of forces act on a rigid body the general effect is to put the body in what is called a *state of stress*. By this is meant that there is a tendency for one part of the body to move relatively to another as a result of the force or forces acting on the body. For example, a colliery winding rope is said to be in a state of stress because the force acting along the rope, due to the weight of the cage, tends to separate the lower portion of the rope from the upper.

As another example we may take the case of an engine crank shaft. Here again, the shaft is in a state of stress because the thrust along the connecting rod, acting at the crank pin, turns the shaft and tends to separate one section of the shaft from another by twisting the one relatively to the other.

Stresses are of different kinds. When the forces act in one straight line in opposite directions, away from each other, as represented by Fig. 1, the stress is known as a *tensile stress*. Thus the colliery winding rope above referred to is subjected to a tensile stress due to the forces acting on it, viz. the weight of the cage and the force or reaction resisting this weight; these forces act in the same straight line but in opposite directions, away from each other. The student is of course aware that wherever there is an action there is an equal and opposite reaction. This is Newton's Third Law of Motion.

Fig. 1

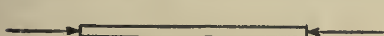

Fig. 2

If the forces act in one straight line but in opposite directions, towards each other, as represented by Fig. 2, the stress produced is known as a *compressive stress*.

A vertical column supporting a roof, for example, is subjected to a stress of this nature, the load resting on the column acting downwards, and the reaction due to the load acting in the same straight line but in the opposite direction, i.e. upwards.

The piston rod of a steam engine is in tension one stroke and in compression the other.

Another kind of stress is that known as *shear stress*. This kind of stress is produced when the forces applied act parallel to each other in opposite directions, as indicated in Fig. 3. The general effect of forces acting on a body in this manner is to cause one portion of the body to slide relatively to another. Metal plates cut in a shearing machine are exposed to shear stress, one portion of plate being separated from another in consequence of two forces acting in opposite directions parallel to each other, one on each blade of the machine.

Fig. 3

An ordinary line shaft on which are secured driving pulleys is exposed to shear stress, but as the force applied tends to twist the shaft, the stress is more often spoken of as a twisting or *torsional* stress.

One other kind of stress sometimes referred to is that termed a *bending* stress. Thus a beam supporting a load in the manner indicated by Fig. 4 is subjected to bending. As a matter of fact, however, although the beam is subjected to bending, the resulting stresses resolve themselves into tensile, compressive, and shear stresses, all three actually existing, as will be shown later, in the beam.

Fig. 4

It must be understood that the stresses we shall be called upon to consider are the result of forces acting external to the body; such stresses are usually known as *external stresses*. It sometimes happens that stresses exist in a body although the body is not exposed to any external forces. Such stresses are termed *initial* or *internal* stresses, and are usually the result of defects in manufacture, such as uneven cooling of metal castings. These initial stresses cannot as a rule be estimated.

2.—Whenever a body is put in a state of stress it undergoes a change of form or shape. This change of form is termed *strain*. A body which was absolutely rigid would of course remain unstrained, no matter how intense was the stress, but no such bodies actually exist. If a body be of a soft and yielding nature, such as indiarubber, the amount of strain for a given stress may be considerable, but if the body be very hard, such as tool steel, it will be strained very little even when exposed to great stress.

The nature of the strain in a body depends on the kind of stress to which the body is exposed. Thus if the stress be a tensile stress, i.e. the result of a simple longitudinal pull, the strain consists of a lengthening of the body concerned, together with a reduction of the lateral dimensions. A bar of indiarubber, for example, when pulled in the direction of its length, and thus exposed to tensile stress, stretches a certain amount, at the same time undergoing contraction in both directions at right angles to the length, and so undergoes a change of form, or is strained.

When a body is exposed to compressive stress, the result of a push or thrust, the strain consists of a shortening of the body, and at the same time the dimensions of the body at right angles to the length, i.e. the breadth and width, increase.

Thus a tensile stress produces an extension or lengthening of a body, with a decrease of lateral dimensions, whilst a compressive stress produces a contraction or shortening of the body, with an increase of lateral dimensions.

In the case of a simple shear stress the strain consists of a distortion or sliding of one portion of the body relatively to another.

When the stress is torsional, as in the case of a line shaft having driving pulleys keyed upon it, the strain may be described as being in the nature of a twist.

With regard to a body subjected to bending, as for example a beam, the effect of the bending is to deflect the beam, so that the strain is in the nature of a deflection.

3.—**Measurement of Simple Tensile and Compressive Stresses.** So far we have dealt only with the meaning of the term stress. We must now consider how the stress in a body exposed to external forces is measured.

When a body is in a state of stress we understand that there is a mutual action between two parts of the body, each part exerting a force upon the other, and thus tending to cause the separation of the one from the other.

Now it is of little use to the designer of a machine or structure to know merely that any particular member of the machine or structure is in a state of stress, as this affords him no indication as to whether or not that member is strong enough to fulfil its function. What he actually wishes to know is the intensity of the stress. *The engineer usually measures stress in pounds or tons per square inch or per square foot of sectional area.* Consider the case of two solid round cast-iron pillars of different diameters, each supporting a certain load. We may assume one column to be of large diameter and the other of small diameter, the former being intended to carry a very heavy load and the latter a comparatively light load. Suppose now we wished to determine which of the two columns was the better able to support the load upon it. (We must assume the columns to be short so that the question of buckling does not enter into the calculation. It may then be taken that each column is subject to pure compressive stress.) In order to do this it would be necessary to reduce the two cases to a common basis of comparison. For instance, it is clearly impossible to say offhand if a column 8 inches diameter, supporting a load of 10 tons, is better able to support its load than a column $2\frac{1}{2}$ inches diameter bearing a load of 1·5 tons. If, however, we reduce the total load in each case to *load per square inch of sectional area*, a comparison can at once be made. For example, the sectional area of a column 8 inches diameter is approximately 50 square inches, and as the total load is 10 tons the load on each square inch of section will be $\frac{10}{50}=\cdot 2$ ton. The sectional area of a column $2\frac{1}{2}$ inches diameter is 4·9 square inches, and as the load supported by this column is 1·5 tons the load per square inch will be $\frac{1\cdot 5}{4\cdot 9}$=approximately ·3 ton. Obviously, then, as the two columns are supposed to be made of the same material, the larger column is better able to support its load without failing than the smaller one.

It is usual, therefore, when determining the strength of any part of a machine or structure to calculate the load per unit area of section, or, in other words, the stress acting over the section.

In the case of the columns above we should say then that the stress in the larger column is ·2 ton per square inch and the stress in the smaller one ·3 ton per square inch.

Sometimes the term *total stress* is used to denote the total load acting on a body, and the term *intensity of stress* to denote the load per unit area of the section, but throughout this work we shall use the terms *load* and *force* to indicate the total load or forces acting on the body externally and the term *stress* to indicate the load per unit area of the section.

Stress may be measured in any convenient units, such as pounds or tons per square inch, kilogrammes per square centimetre, tons per square foot. In engineering work, and particularly in machine design, the usual units are pounds or tons for the loads, and square inches for the sectional areas.

4.—**Ultimate or Breaking Stress.** Having determined the stress in any particular case, the student will naturally wish to know of what use this is in determining whether or not the member concerned is sufficiently strong to sustain safely the load imposed upon it, and this we may now proceed to explain.

Every metal used in the construction of machinery has been carefully tested with the object of determining the stress which would cause it to rupture. This stress is termed the *ultimate stress* or the *breaking stress*. Mild steel, for example, has an ultimate stress of about 30 tons per square inch in tension, which means that a bar of mild steel one square inch in sectional area would just fail under a tensile load of 30 tons. A mild steel bar of 2 square inches sectional area would break with a load of twice 30 tons, or 60 tons.

A cast-iron bar of 1 square inch section would break with a tensile load of about 7·5 tons, so that the ultimate tensile stress of cast iron is about 7·5 tons per square inch.

The same cast-iron bar, if exposed to a purely compressive load, would require a load of about 45 tons to cause failure. Cast iron is therefore said to have an ultimate compressive stress of about 45 tons per square inch.

The figures quoted refer, of course, to average samples of mild steel and cast iron. No two samples of either metal would be found to possess exactly the same ultimate stress, as all samples differ somewhat in structure and composition. In fact, the ultimate stress of mild steel in tension varies from 27 to 33 tons per square inch, and in the case of cast iron in tension the ultimate stress is from 5 to 15 tons. The figures above quoted represent average values.

If, then, it is known what load per square inch of section is required to break any particular metal and we calculate the stress actually imposed on any member of a machine or structure composed of this metal, we can at once say whether or not the member is likely to fail.

Take again the case of the cast-iron columns above referred to. It was found that the larger column was exposed to a stress of ·2 ton per square inch and the smaller one to a stress of ·3 ton. Now average cast iron has an ultimate compressive stress of about 45 tons per square inch, so both columns are amply strong so far as regards purely compressive stresses. We have assumed the columns to be short, as otherwise they would be liable to buckle and their true strength could not then be determined in the simple manner explained.

5.—**Factor of Safety.** If some member of a structure be exposed to a stress somewhat less than the ultimate stress of the material of which the member is composed, it may be supposed that the member is quite safe to carry the load imposed upon it. It would of course appear that failure could not occur if the actual stress never reached the ultimate stress. In actual practice, however, great care is always taken to ensure that the working stress shall never be nearly so great as the ultimate stress. Thus it is customary for the designer of a machine to proportion the various parts in such a manner that the working stress will only be a certain fraction of the ultimate stress. In this way a certain margin of safety in working is ensured. The ratio of the ultimate stress to the working stress is termed the *factor of safety*.

The reason for adopting a factor of safety will be evident from the following considerations.

As already pointed out, different samples of the same material may vary considerably in structure and composition, and consequently in strength. For instance, we have seen that the tensile strength of cast iron varies from as little as 5 tons to as much as 15 tons per square inch, whilst that of mild steel varies from 27 tons to 33 tons per square inch. There is thus an element of uncertainty in the actual strength of the materials used, although this need not be of great importance in the case of mild steel or wrought iron if due care be exercised in the process of manufacture.

Again, in the construction of the machine or structure, there is always the possibility of bad workmanship entering into the question, in consequence of which the actual strength of the parts may be more or less seriously reduced.

Another factor to be considered is the deterioration or weakening of the materials which may take place as time goes on from various causes.

A further factor, and one of the greatest importance, is the manner in which the loads to be sustained act on the machine. If the loads be steady and uniform we can generally calculate the stresses due to them, but otherwise it may be impossible to determine them with any degree of certainty.

It is customary to class the loads acting upon a machine or structure as *dead loads* or *live loads*. A *dead load* is one which produces a constant stress or a stress which increases or decreases very gradually.

A *live load* is one which is applied more or less suddenly, or one which varies from instant to instant.

The load on a road bridge, due to the weight of the roadway, is an example of a dead load, whilst the load due to the weight of a train running rapidly over a railway bridge is a live load.

The value of the factor of safety to be adopted in any given case naturally varies in accordance with the conditions. Thus if the materials used in the construction are reliable and the loads steady, a lower factor may be used than would be required if the materials were not reliable and the loads not steady.

In actual practice the factor of safety is seldom less than three, i.e. the working stress is seldom more than one-third the ultimate stress; in some cases it is as much as twelve or even more. As an example of a structure where a comparatively low factor of safety may be adopted we may mention the steam boiler. Here the material from which the structure is made, viz. mild steel, is very reliable, its strength being known with certainty, the forces acting on the structure may also be calculated accurately, and the forces are extremely steady. Further, if the conditions of working be satisfactory, deterioration may be almost entirely prevented. As an example of a structure where a high factor of safety is necessary a crane may be cited. Here, although the material may be of the best quality, the actual loads may be applied in the nature of shocks, so that they are, strictly speaking, indeterminate; further, repeated application of unsteady loads may impair the quality of the material as time goes on, and it is most important therefore that a high factor of safety should be adopted.

6.—**Stress-Strain Relations: Elasticity.** We have seen that strain is a deformation of shape produced in a body by the action of stress.

The deformation so produced may be one of two kinds, viz. *elastic* or *plastic.* If, when the load which gives rise to the stress is removed, the body regain its original shape, or, in other words, if the strain entirely disappear on removing the load, the body is said to be *elastic.* If, on the other hand, the body retain its deformed shape after removal of the load, it is said to be *plastic.* A piece of indiarubber after being stretched regains its original shape when the stretching force is removed and would therefore be classed as an elastic body. A piece of putty or clay after being deformed does not recover its original shape at all and would therefore be classed as a plastic body. All the materials of engineering possess the properties of elasticity and plasticity more or less, but none is absolutely elastic or absolutely plastic. Both properties may of course be useful. Thus in machine work the property of elasticity is required in the material composing the different members of the machine, as without it each member exposed to stress would be permanently strained every time the load came upon it. An example of the use of plasticity is seen in the case of mild steel, which may be drawn into wire or forged into various forms, and so on.

When a body after being strained does not recover its original shape on removal of the load it is said to take on a *permanent set,* the set being that part of the strain which does not disappear.

The materials mostly used in the construction of machines—iron and steel, for example, possess the property of elasticity in a high degree, so far as small or moderate stresses are concerned.

For instance, a bar of steel if strained by a tensile load which does not stress it beyond a certain limit quickly regains its original form when the load is removed. If, however, the bar be stressed beyond the limit referred to some of the strain will not disappear when the load is removed, and the stress in the material when the bar thus begins to take a permanent set is termed the *elastic limit* of the material.

If, then, a body be stressed beyond its elastic limit it will begin to suffer permanent deformation, and it is obvious therefore that the designer of a machine should take care that the stress in every member of the machine be kept well below this limit. As a matter of fact the elastic limit is a most important factor in determining the safety of a material, and for this reason it would perhaps be better when fixing upon the stresses to be allowed in any given case if regard were had to the elastic limit of the material concerned rather than to the ultimate stress.

Suppose, for example, it were decided to adopt a factor of safety of three when designing a certain piece of machinery. If it so happened that the elastic limit of the material were one-third the breaking stress, the machine, although being worked at a stress of only one-third the ultimate stress, would still be stressed up to its elastic limit, and for this reason would not be safe.

The elastic limit in tension for the various qualities of iron and steel is known, and in order to give some idea as to what relation this bears to the ultimate stress, it may be mentioned that for wrought iron and mild steel the elastic limit is rather more than one half the ultimate stress.

In the case of cast iron the elastic limit is not clearly defined; this metal is indeed very wanting in the property of elasticity, and under the smallest loads, it appears that a slight amount of permanent set is produced. The elastic limit of cast iron is frequently assumed to be about one-third its ultimate strength.

7.—**Measurement of Simple Tensile and Compressive Strains.** When we speak of a body being strained we usually understand that the body has undergone a deformation of shape through being subjected to a stress. In the case of a rod of iron or steel elongated a certain amount by a tensile load, we might say that the rod had been strained to the extent of some fraction of an inch. In dealing with the measurement of stresses, we saw that before we could compare the strengths of two columns of different diameters supporting different loads we must reduce the results to some standard or basis of comparison.

Similarly, we must have a common basis of comparison for measuring strains. Thus if a certain bar be stretched $\frac{1}{2}$ inch by a certain load and another bar of the same diameter but of twice the length be stretched the same amount, then although the extension is the same in the two cases we could not say the two bars had been strained equally, because with the same force and the same diameter of bar, the extension is greater the greater the length of the bar. The element of length therefore enters the question, and for this reason we must take it into account when measuring strains. Hence, except when we are merely speaking of a strain in general terms, we understand the term "strain" to mean *elongation per unit length. The actual strain in a stretched bar is consequently equal to the extension divided by the original length of the bar.*

It is important to notice, then, that strain, strictly speaking, is not extension; the latter is an increase of length whilst the former is a fraction or ratio.

8.—**Laws governing the Amount of Extension of a Stretched Bar: Hooke's Law.** An important relation between stress and strain was discovered by Robert Hooke so far back as 1676. Hooke found that *the strain produced by a stress is directly proportional to the stress, providing the limit of elasticity be not exceeded.* This law is known as Hooke's law. Thus if the stress be doubled the strain will also be doubled. If the stress be increased beyond the elastic limit the strain increases more rapidly than it did before the limit was reached, and Hooke's law no longer applies.

Hooke's law applies to any one kind of stress—tensile, compressive, or shear stress.

In addition to the important law above defined there are two other laws which govern the amount of extension of a bar subjected to tensile loads within the limits of elasticity. The first of these is that *the extension is directly proportional to the length of the bar.* Thus if the length of the bar be doubled the extension will also be doubled if the force applied and the diameter of the bar be the same as before. According to the second law *the extension is inversely proportional to the sectional area of the bar.* That is to say, if we take two bars of the same length and material and subject them to the same pulling force, then if one bar be twice the sectional area of the other it will only be stretched one half the amount which the other will be. This seems obvious from the fact that the larger bar offers twice the amount of material to resist the extension as does the smaller.

9.—**Young's Modulus of Elasticity.** According to Hooke's law the strain produced in a bar subjected to tensile loads is directly proportional to the stress which produces it if the limit of elasticity be not exceeded.

It follows from this that the ratio of the stress to the strain in any particular material will be a constant quantity within the limits referred to. This ratio is known as *Young's Modulus of Elasticity.*

Thus, $$\text{Modulus of Elasticity} = \frac{\text{Stress}}{\text{Strain}}.$$

The modulus of elasticity of a material is a very important constant and frequently enters into the calculations of the engineer and machine designer ; for this reason the student should possess clear ideas as to the exact and full meaning of the term.

A little consideration will serve to show that the modulus of elasticity of a material is the stress which would produce unit strain, assuming Hooke's law to hold throughout ; in other words, it is that stress which would double the length of a bar on the assumption that the elongation was proportional to the stress throughout and that it was possible to stretch the bar by an amount equal to the original length of the bar.

As a matter of fact it is quite impossible to stretch any bar to double its length, because every material used in the construction of machinery would rupture long before it could be elongated to such an extent; the modulus of elasticity of a material is therefore an imaginary quantity, but it is nevertheless of great use, as shall be subsequently seen.

We have seen that a stress is usually specified as so many pounds or tons per square inch of sectional area and that a strain is merely a fraction or an abstract number. As the modulus of elasticity is equal to the stress divided by the strain it will be clear that this constant is also specified in pounds or tons per square inch of sectional area of the material.

10.—**Weakening due to repeated Applications of Stress: Fatigue.** Experience has shown that when a piece of metal is subjected to a large number of repetitions of stress it tends to become gradually weaker. Suppose, for example, a bar of mild steel be placed in a tensile testing machine and pulled repeatedly with a load less in amount than the load required to break it. Then it will be found that in consequence of the repeated applications and removals of the load the material has become weakened in some manner, and it will be possible to break the bar eventually with a load less than the original breaking load.

The weakening which bodies undergo as a result of repeated applications of stress is commonly known as "fatigue."

The extent to which weakening occurs depends on the magnitude of the stresses, or on the range through which the stress varies. If the stresses lie well within the elastic limit an indefinite number of applications may fail to produce any appreciable weakening, but if they approach the ultimate stress of the material, comparatively few applications may produce rupture. If the stresses be alternately tensile and compressive the tendency to weaken will be greater than if the stresses be all of one kind of the same magnitude, because the range of stress in the former case is double what it is in the latter case.

When a piece of iron or steel has become weakened by fatigue the weakness may be corrected by *annealing*. This consists in passing the weakened piece through an oven or furnace and then allowing it to cool gradually. By so doing, the structure of the material becomes rearranged and its strength more or less restored. Chains, crane hooks, etc., are annealed periodically for this reason.

11.—**Tensile Test of a Piece of Iron or Steel.** With the object of giving the student clear ideas in regard to stress, strain, and elasticity we shall now describe somewhat briefly how a piece of iron or steel behaves when subjected very gradually to increasing tensile loads, the loads commencing from zero and being increased until the piece breaks.

If the piece to be tested is merely a wire a tensile test can be very simply carried out by fixing one end of the wire to some convenient point overhead and then adding weights to the other end. The extensions due to the loads may be measured by means of a vernier attached to some convenient part of the wire.

When, however, the piece to be tested is a bar instead of a wire a more elaborate testing arrangement is required. In such cases a special machine, known as a *testing machine*, is employed.

There are many varieties of tensile testing machines in use at the present time, but the principle of action is much the same in all of them. The specimen piece is held firmly at each end in suitable shackles, one of which is connected with a hydraulic plunger or ram, whilst the other is connected with a lever and weight arrangement.

In order to apply the tensile loads to the specimen, hydraulic pressure is applied to the plunger or ram, and as the pressure can be applied and increased very steadily the specimen is not subjected in any way to live loads. By means of the lever and weight arrangement the actual load put on the specimen can be readily measured.

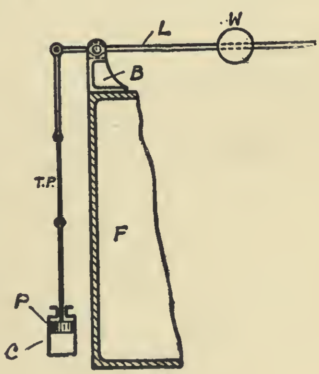

Fig. 5

A general idea of the design of a testing machine will be obtained from a study of Fig. 5, which represents diagrammatically a simple machine of the vertical type.

The essential parts of the machine are the main frame F, a hydraulic cylinder C, attached to the frame, the plunger P with its rod, a bracket B on the top of the frame, and a lever L with its jockey weight W. The test piece, marked TP, is seen to be connected at its lower end through the plunger rod with the plunger P, and at its upper end with the lever L. When the hydraulic pressure is applied, the lever, the fulcrum of which is in the jaws of the bracket B, is maintained in balance by moving the jockey weight W along the lever arm. The position of the weight at any particular instant thus represents the total pull or tensile force acting on the specimen, regard being had of course to the amount by which W is multiplied as a result of the mechanical advantage of the lever. Thus if W be 100 lbs. and the distance from the fulcrum to W be five times the distance from the fulcrum to the left end of the lever the total pull on the specimen will be five times 100 lbs. or 500 lbs.

The extensions due to the loads may be measured from time to time as the test proceeds by means of a divider and a steel rule, but if very accurate measurements be required, it is usual to employ a special apparatus called an *extensometer*.

Let us consider now how the piece behaves as the load is applied and gradually increased.

Starting with no load on the piece, the usual plan is to turn on the water pressure until a certain small load has been applied. The operation is then stopped whilst the load and the corresponding extension are carefully measured and tabulated. The pressure is then again turned on and the load increased by the same amount as before, when the operation is again stopped, the load and the extension measured, and the results tabulated. This procedure is continued until the specimen breaks, the loads and the corresponding extensions being measured and tabulated at intervals throughout the test.

In carrying out a tensile test of this nature, it would be observed that during the early part of the test the extensions were very small — so small, in fact, that they could not be measured with any degree of accuracy by a divider and steel rule.

It would also be found that the extensions were approximately proportional to the loads or the stresses, in accordance with Hooke's law, during this portion of the test, and if the load were removed the piece would return to its original length.

After a time, a point is reached when the above conditions no longer obtain. The extensions will now begin to be greater for the same increase in the load than formerly, and if the load be removed the extension will not entirely disappear, i.e. the specimen will have commenced to take a permanent set. The point referred to represents, of course, the elastic limit of the material.

Shortly after the elastic limit is passed, another point, termed the *yield point*, is reached. At this point the specimen draws out considerably, the extensions now being many times the magnitude they were previously. The extensions are indeed so great at this stage that they may be easily measured by means of a divider and a steel rule.

As the load is still further increased, the piece continues to extend until rupture takes place. A peculiar action takes place towards the end of the test. At a certain point it is found that the bar continues to extend even if the load should be no longer increased, and not only this, but the load may be actually reduced and the extension will continue until the bar breaks. The explanation of this peculiar action lies in the fact that when the maximum load is reached, the diameter of the bar at a certain part of its length commences to be reduced in a very marked manner. The cross sectional area of the bar at this particular point therefore is also rapidly reduced, and consequently, although the actual load may not be increased, the load per unit area or the stress, on which the strain really depends, is considerably increased, with the result stated.

The behaviour of the bar during the test may be shown diagrammatically by plotting the results on squared paper, the loads being plotted as ordinates and the extensions per unit length as abscissae. Such a diagram is known as *a load-strain diagram.*

In Fig. 6 is shown one of these load-strain diagrams.

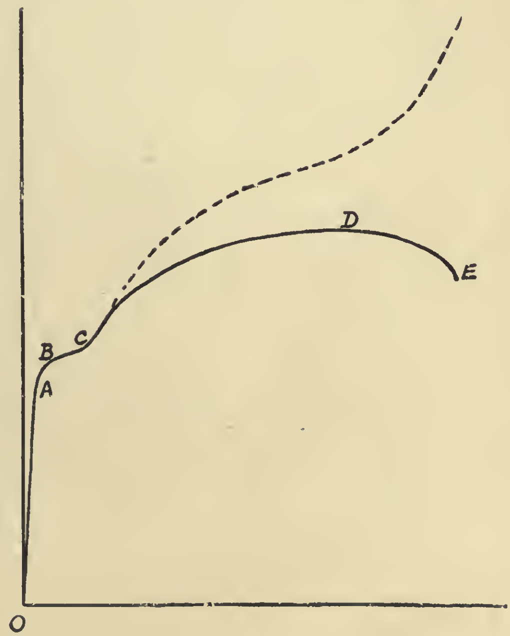

Fig. 6

Referring to the figure, it will be observed that the first portion of the diagram, O to A, is straight, thus showing that up to the point A the strain is directly proportional to the load, or the bar is elastic. It will also be observed that the strain is very slight up to the latter point, although the load is comparatively great. After point A there is a deviation from the straight line up to point B, the extensions increasing more rapidly. At this latter point the extensions are seen to increase rapidly, although the loads increase only slightly, and this continues until the point C is reached. The point B represents the yield point. After point C the curve rises more steeply, the extensions being less for the same increase of load than just before ; the curve, however, gradually becomes less steep until the point D is reached, thus showing that the extensions are again becoming greater for the same increases in the load.

At point D the maximum load is reached, and from this point it will be noticed that the extensions increase although the loads actually become less. The explanation of this has already been given.

At the point E rupture of the specimen takes place.

When the tabulated results taken during the tensile testing of a bar of iron and steel are plotted on squared paper, the loads as ordinates and the extensions as abscissae, the resulting diagram or curve is, as already pointed out, a *load-strain* diagram. It is important to note that such a diagram is not a *stress-strain diagram* (i.e. a diagram in which the stresses are plotted as ordinates and the strains as abscissae), although it is frequently spoken of as such. To plot a stress-strain diagram, each of the load observations would require to be divided by the least sectional area of the specimen at the time the observation was taken. If the sectional area of the specimen did not alter during the loading the stress would be proportional to the load, and a load-strain diagram would also represent a stress-strain diagram, proper regard of course being had to the change of scale in the two cases. We know, however, especially in the case of ductile materials, such as wrought iron and mild steel, that the sectional area alters very materially during the later applications of the loads, so that for these materials a stress-strain diagram would differ considerably from a load-strain diagram.

During the early portion of a tensile test there is little alteration in the sectional area of the test piece, and the load-strain and stress-strain diagrams therefore coincide. In order to make matters clear, the stress-strain diagram for the test represented by Fig. 6 is shown dotted, the load-strain diagram being shown in full. It will be observed that the two curves coincide until after the point A, the elastic limit, is passed. After that point the sectional area of the bar is appreciably reduced, so that the value of the load divided by the new sectional area, i.e. the stress, is increased. The sectional area now decreases more and more, and the stress-strain curve consequently rises more and more above the load-strain curve, the rise being very rapid after the maximum load has been applied, when the latter curve commences to fall.

If a stress-strain curve were required, it would be necessary to gauge the least lateral dimensions of the bar in addition to the extensions each time records were taken, and to calculate the sectional area and then divide the loads by the areas in order to get the stresses.

12.—**Apparent and Actual Stress.** When determining the magnitude of the stress in a loaded rod or bar, it is customary to ignore the alteration which takes place in the sectional area of the bar, the stress being obtained by dividing the load by the *original* cross sectional area.

From what has been said in the previous article, it will be understood that this is not strictly correct. The stress obtained by dividing the load by the original cross sectional area is sometimes termed the *apparent stress* to distinguish it from the *actual stress*, which of course is equivalent to the load divided by the new sectional area.

The apparent stress is, however, as already stated, always taken in actual practice, and when it is considered that the different members of a structure are never stressed so severely as to suffer appreciable strain or deformation, it is obvious that there is little necessity to take into account the very slight alteration of sectional area which does actually take place.

So far as the ultimate or breaking stress is concerned, the alteration in sectional area is considerable, but here again the apparent stress is that which is referred to, this being equal to the maximum or breaking load divided by the original sectional area. It might be thought that, in specifying the ultimate stress, it would be better to give the actual stress, but it must be remembered that, although the section is very much reduced at the point where breakage occurs, the actual load on the specimen at the time of rupture is also much less than the maximum load (as explained in connection with the tensile test of a piece of iron or steel), and the actual stress does not therefore differ so much from the apparent stress as would at first sight appear.

Chapter II

PROPERTIES OF IMPORTANT ENGINEERING MATERIALS

13.—BEFORE going into the question of strengths, it will be advisable to explain more or less briefly the characteristic properties of some of the principal materials used in the construction of machinery and engineering structures generally.

An elementary knowledge of the properties of the various materials enables the designer to decide which particular material is the best to use for any particular purpose. An elementary knowledge of the principal processes by which iron and steel are produced is also essential to every engineer, as in specifications of engineering designs and structures it is frequently stated by which particular process the material to be employed is to be made.

Certain terms relating to the properties of materials are constantly being used by engineers, and the exact meaning of these terms must therefore be explained at this stage.

Tenacity.—This term is applied to denote the resistance to fracture which a body offers when subjected to a pulling or stretching force. Generally speaking, all the metals possess this property in a greater or less degree, and the property is a most important one so far as regards the materials used in the construction of machinery.

Hardness.—A body is said to possess the property of hardness, or to be hard, when it offers considerable resistance to being indented by impact or pressure with another body, or to being worn by friction with another body. Thus machine cutting tools are made hard to prevent them from being blunted by contact with the materials they are intended to cut.

Softness.—This is, of course, the converse of hardness.

Brittleness.—When a body breaks readily on being subjected to shocks, it is said to be *brittle*. Glass, for example, is a brittle material, as it is easily broken by being struck, or by being dropped on the floor.

Ductility.—This is the property possessed by certain bodies of being so constructed that they may be drawn out in the direction of their length, the elongation being permanent.

Materials used in the manufacture of wire must possess the property of ductility in a high degree. The process of wire-making consists essentially of drawing round rods through holes in a steel-faced plate, the diameter of the first hole being slightly less than the diameter of the rod, and repeating this operation with smaller and smaller holes, until eventually the rod is drawn out into wire of the particular diameter required. The material tends to become hard and brittle as a result of being drawn out in this manner.

The most ductile metals are those which possess the properties both of tenacity and softness.

Malleability.—A body is said to be malleable when it can be beaten out and extended in all directions.

Welding Power.—Separate pieces of certain metals, when heated to a high temperature, may be joined together by hammering so as to form one piece. Such metals are said to be *weldable.*

14.—**ENGINEERING MATERIALS.** The materials principally used in engineering work are cast iron, wrought iron, steel, copper, alloys such as brass and bronze, consisting of two or more metals melted together, timber and masonry, the first three being those which mostly concern us from the machine construction point of view.

15.—**Cast Iron.** The various forms of iron and steel are all derived from iron ores, which consist of iron in combination with certain other elements, principally oxygen and carbon, in the form of oxides and carbonates.

In order to extract the pure metal from the ore, it is necessary to smelt the ore in a special furnace known as a "blast furnace."

The operation of smelting consists of blowing a strong blast of air on to the fuel in the furnace to generate an intense heat sufficient to melt the iron, and then adding what is called a "flux," the object of which is to combine with the impurities in the ore and to facilitate their fusion. The flux unites with the impurities, forming slag, and thus setting the greater part of the iron free. The latter, as it melts out, falls to the bottom of the furnace, whilst the slag, being lighter, rests on the top of the molten iron.

When a sufficient quantity of the molten metal has collected in the bottom of the furnace, the latter is tapped, and the iron allowed to flow out into a long channel formed in a bed of sand. A number of smaller channels are connected at right angles with the main channel. It is customary to speak of the main channel as the "sow," and the branch channels as the "pigs."

The iron, after running down the main channel, and along the "pigs," is allowed to cool, and as it cools it solidifies into castings, the form of which corresponds with the shape of the "pigs."

These castings are known commercially as *pig iron*, and it is in this form that the metal is supplied to the ironfounder or the iron manufacturer. By subjecting this pig iron to certain processes, which will be described only very briefly, cast iron, wrought iron, and steel are produced.

During the process of smelting, the molten iron takes up from the fuel, etc., small percentages of a number of other elements, such as carbon, silicon, sulphur, phosphorus, and manganese. All these elements affect, in some way or another, the properties of the metal, but the element which plays the most important part is carbon. We shall see shortly that the principal differences between cast iron, wrought iron, and steel depend for the most part on the percentage of carbon contained.

Pig iron is frequently classed as cast iron, but the real cast iron, as used for producing castings of any shape desired, is obtained by melting down the pig iron again in special furnaces known as "cupolas." A cupola is similar to a blast furnace, and the action is also similar. A little limestone is usually added as a flux with the object of removing some of the impurities which would still otherwise remain.

The pig iron obtained from the blast furnace varies considerably in character, and for commercial purposes it is divided into seven or eight varieties, classed as No. 1, No. 2, etc., up to No. 8; but it is not necessary here to consider the characteristic properties of these different varieties, as we are more concerned with the real cast iron as used for making special castings.

Just as there are different varieties of pig iron, so there are different varieties of cast iron.

Engineers and ironfounders usually divide the different varieties of cast iron into three important classes, viz. *grey*, *white*, and *mottled* cast iron, according to the appearance presented by a fractured surface of the iron.

It has been already pointed out that the iron in the blast furnace takes up other elements, of which the most important is carbon. The total amount of carbon in cast iron varies from about 2 to 5 per cent. A certain portion of this carbon is chemically combined with the metal, whilst the remainder exists in the free state as graphite, i.e. is simply mechanically mixed with the iron.

Whether any particular variety of cast iron is grey, white, or mottled depends upon how the carbon exists in the iron. If most of the carbon exists in the free state, the fracture of the iron presents a dark grey colour, and hence such iron is known as *grey* cast iron.

If, on the other hand, all the carbon is chemically combined with the metal, the fractured surface presents a silvery white appearance; hence the reason for calling the iron *white* cast iron.

When the carbon is divided nearly equally into the combined and the free state, the fractured surface presents a mottled appearance, white with grey patches, or grey with white patches; for this reason such iron is called *mottled* iron.

The larger the proportion of the carbon chemically combined, the harder and the more brittle is the iron. Hence the white varieties of cast iron are more brittle and harder than the grey varieties.

Cast iron is of great value to the engineer, owing to the fact that it can be melted down and run into moulds. It is thus possible to make an object of almost any shape desired by preparing a suitable mould. The moulds are prepared in sand, from wooden, or in some cases metal, patterns of the particular object required. Owing to the fact that cast iron contracts on cooling, it is necessary to make the patterns slightly larger than the objects to be cast. The usual allowance is about one-eighth of an inch per foot in every direction.

When designing and making a casting the greatest care must be taken to guard against internal stresses, which may seriously weaken the object and thus make it a source of danger. Internal stresses are mostly indeterminate, and for this reason alone they should be eliminated as far as possible. To this end the designer must exercise skill in arranging the design to the best advantage, whilst the moulder must arrange the mould in the best possible manner, taking particular care to avoid sudden cooling down of any portion of the casting.

Internal stresses in castings are caused through some portions of the casting cooling down and consequently contracting more quickly than other portions.

The object of the designer should be to design the casting in such a manner that all parts will cool down at as near the same rate as possible. It is not difficult to understand that if a thin portion adjoins a thick portion of metal, the thinner portion will cool more rapidly than the thicker one. When the one has cooled down, contracted, and become more or less solid, it tends to resist the contraction of the other, whilst the latter, when it contracts, tends to compress the portion which has already contracted. In this way the casting becomes stressed and is consequently weakened before being exposed to the action of the loads it is really intended to sustain.

The designer should therefore, as far as practicable, endeavour to avoid having a thin portion immediately adjoining a thick portion. Where this cannot be avoided, much may be done to guard against internal stresses by making the change in section as gradual as it is possible to make it. Abrupt changes of size, sharp corners, etc., are particularly objectionable, and are a source of serious weakness. When the molten cast iron is poured into the mould and commences to cool, the crystals of iron arrange themselves along lines of flow ot heat, and in the case of a sharp corner, continuity of the arrangement of crystals is broken, with the result that there is a line of weakness at the corner, the weakness being more serious the sharper the corner.

All corners should consequently be rounded off with as large a radius as possible.

A cast-iron belt pulley is an example of a casting in which internal stresses are very liable to be set up. The rim is thin and possesses large cooling surface, so that it cools much quicker than the arms which connect it with the boss of the pulley; the rim consequently becomes rigid sooner than the arms, and when the latter cool they contract and so impose a stress on the rim. If the latter be extremely thin and the arms very heavy, the straining may be so severe as to cause fracture near the junction. The tendency to fracture is of course increased if the cooling down be hastened at all.

Indeed, it is no unusual occurrence for a casting which has been cooled down quickly to split up into two or more portions shortly after it has been cast.

For general casting purposes the grey varieties of pig iron are mostly used, because, although they require a higher temperature to melt them than the white iron, they are more fluid in the molten state, and they expand slightly on solidifying, thus filling up the moulds properly and so producing well-defined castings.

For certain purposes it is of great advantage to have the surface of a casting hardened. For instance, a surface constantly exposed to wear would resist the wear for a much longer period if it were made hard than it would if left soft. In such cases the castings are cooled down comparatively rapidly at the surfaces which require to be hardened. This is effected by lining the mould at the part required with cast iron, the latter being protected by a coating of loam. As cast iron is a comparatively good conductor of heat, the lining thus conducts away the heat from the molten metal in contact with it, the effect of which is to convert the soft grey iron usually used for casting purposes into a hard white cast iron. The finished casting consists of a body of grey cast iron (which is soft and tough and the best variety for resisting stresses) with a hardened outer surface of white iron. Such castings are known as *chilled castings*.

Owing to its brittle nature cast iron is not a good material to resist shocks, and for this reason parts of machinery which are exposed to shocks are generally made of a more malleable material such as wrought iron or steel. It is, however, practicable to impart the useful property of malleability to cast iron by a certain process. This process consists in embedding the finished casting in a box, surrounding it with an oxide of iron (generally powdered red hematite), and heating it for a prolonged period out of contact with the air. The effect of this is to remove a portion of the carbon contained in the iron and so convert the metal into more or less pure wrought iron, which is much more malleable, tougher and stronger than cast iron. Castings made in this manner are known as *malleable castings.*

The fact that ordinary castings can be rendered malleable is of very great use in the engineering industry. When a piece of machinery is required to be malleable and tough it is generally made of wrought iron or mild steel, but intricate designs cannot be made very well of these materials, and cast iron (or cast steel) must be used in such cases. Ordinary cast iron, however, would be unsuitable on account of its brittle nature, but the required design may be made in cast iron and then rendered malleable in the manner explained. Malleable castings thus combine to a large extent the advantages possessed by cast iron, as regards the possibility of their being made of any desired form, with those possessed by wrought iron and mild steel, viz. toughness and strength.

The extent to which a casting is rendered malleable naturally depends upon the thickness of the casting and the time of exposure. A thin casting may be malleable throughout, but a thick one will only be malleable to a certain depth, the interior metal being unaltered.

In addition to containing carbon, cast iron contains certain small percentages of other elements, including silicon, manganese, phosphorus, sulphur, etc. The various elements, as already stated, each affect in some way or another the character of the iron.

Thus silicon renders the iron more fusible, but makes it more brittle, and thus liable to fracture from shock. Manganese tends to whiten the iron, and if present in too great a quantity makes the metal hard and brittle.

Phosphorus makes cast iron more fusible and more fluid in the molten state. Its presence is therefore desirable in those varieties of cast iron used in the manufacture of light ornamental castings. This element is, however, objectionable on the grounds that it makes the metal brittle.

Sulphur is an objectionable ingredient, as it tends to reduce the strength of the iron and to render the latter hard.

16.—**Strength of Cast Iron.** The strength of cast iron varies within large limits. The average ultimate tensile stress of this metal may be taken to be 7·5 tons per square inch. Inferior qualities of cast iron may have an ultimate stress of only 5 tons per square inch in tension, but in some qualities the strength is three times this figure, whilst in a few instances the figure has reached as much as 19 tons to the square inch.

Cast iron is exceptionally strong in compression, and is generally considered to be six times as strong in compression as in tension, its average ultimate compressive stress being 45 tons per square inch. The ultimate compressive stress, however, varies from as little as 22 tons for the inferior qualities to as much as 60 tons per square inch for the best qualities.

The average shearing strength of cast iron is usually greater than its average tensile strength; it is often taken to be about 11 tons per square inch. With some specimens, however, it has been shown to be weaker in shear than in tension.

It has been previously pointed out that the elastic limit of both wrought iron and mild steel is approximately one half the ultimate stress. In the case of cast iron, however, the elastic limit or the elastic strength is not clearly defined, but it is often assumed to be only about one-third the ultimate strength.

17.—**Commercial Tests for Cast Iron.** With the object of ascertaining whether or not a casting is sound and of good quality, certain tests are sometimes carried out. Much of course can be done by visual examination. Thus a good casting will present a smooth exterior surface and will be free from blow-holes, honeycomb, or other defects; the various edges will be sharp and well defined, whilst the texture will be close grained and of a uniform character.

The test mostly applied to cast iron consists in loading transversely rectangular test bars which are either cast along with the main casting or cast separately from the same metal. The sizes of the test bars are generally 1 inch wide by 1 inch deep by 12 inches long, and 1 inch wide by 2 inches deep by 36 inches long. The bars are supported at their ends and then loaded with a certain weight applied at mid-length. This weight should be sustained for a certain length of time, one hour or two hours for example.

In addition to sustaining the weight without breaking, the test bar should show a certain amount of deflection at its centre, as this tends to show that the iron is not too hard and brittle.

In the case of castings which are to be subjected to internal fluid pressure, such as steam pipes, engine cylinders, etc., it is usual to apply a hydraulic test, the castings being subjected to a water pressure of from two to five times the actual working pressure.

18.—**Wrought Iron.** As previously pointed out, the principal difference between cast iron, wrought iron, and steel depends mostly on the amount of carbon contained in the metal. Cast iron, as we have seen, contains from 2 to 5 per cent. of carbon; wrought iron contains very little carbon, usually not more than about ·15 per cent. Wrought iron is indeed almost pure iron.

It will be understood, then, that if the bulk of the carbon contained in cast iron could be removed by some process, wrought iron might be produced from cast iron.

As a matter of fact, it is possible to remove the carbon from cast iron, and wrought iron is generally made from cast iron in this way.

A number of processes are required in the manufacture of wrought iron. The main process is that known as *puddling*. In this process the cast iron is melted in a special furnace known as a reverberatory furnace, and the metal is exposed to the action of a strong current of air and is well mixed with oxidising substances. The effect of this is to cause oxidation and consequent removal of the carbon and other elements (silicon and manganese, etc.).

As the carbon is removed, the metal becomes less fusible, and it is eventually removed from the puddling furnace in soft spongy masses which have been worked into large balls.

The next process is that of *shingling*, which consists of hammering and squeezing the balls with the object of removing the slag mixed with the metal. The slag is largely removed during the shingling process, the balls being gradually converted into rectangular blocks termed *blooms*.

The blooms are now rolled into bars known as "puddled bars" by passing them between grooved rollers.

These puddled bars represent wrought iron of an inferior quality only, as they are not homogeneous in character, whilst they still contain a certain amount of slag which has not been removed during the shingling process. They have, consequently, to be subjected to further processes, viz. *piling*, *reheating*, and *rolling*. Thus the puddled bars are cut into suitable lengths and arranged in piles, which are heated to a welding heat. When thoroughly heated the piles are withdrawn, the bars are welded together, and then passed through more rolls. After this treatment the bars, which are now known as "merchant" bars, are much stronger than previously, but the iron is still of inferior quality.

The quality of the metal may be improved by repeating the processes of piling, reheating, and rolling once, twice, thrice, or even four times. By repeating the processes once, "best" bar is produced, and by repeating twice, "best best" bar is obtained. "Best best best," or "treble best" bars are obtained by repeating the processes three times, i.e. by piling, reheating, and rolling four times.

19.—**The Characteristic Properties of Wrought Iron** are ductility and malleability, toughness and strength. The purer the metal, the more marked are these properties. Wrought iron is particularly suited to the manufacture of such parts of machines and structures as are required to sustain heavy shocks, as, for example, connecting rod bolts, chains, etc.

Another useful property possessed by wrought iron is its suitability for welding. When heated to a sufficiently high temperature the metal becomes more or less viscous, and two or more pieces in such condition may be united or *welded* together by hammering. The numerous links which make up a wrought-iron crane chain are formed from single pieces of round iron which are bent to the required shape and then closed by welding the ends together.

Just as it is desirable to have a hardened outer surface in the case of certain castings, so it is in the case of wrought-iron forgings. The outer surface of a wrought-iron body may be hardened by heating the body in contact with substances rich in carbon, such as bone dust. By so doing the carbon in the bone dust is taken up by the iron, and the latter is thus converted into steel at the surface. The steel may then be hardened by immersing the body in water. This process is known as *case-hardening*.

As in the case of cast iron the quality of wrought iron may be affected by the presence of small quantities of certain elements in the iron. The presence of sulphur, for example, tends to make the metal liable to crack if bent or worked at a red heat; this defect is termed *red short*. The presence of phosphorus in wrought iron tends to make the metal brittle when cold; this defect is termed *cold short*.

20.—**Strength of Wrought Iron.** The tensile strength of wrought iron varies within fairly large limits, and depends largely on the extent to which the metal has been worked. Thus, repeated forging increases the strength up to a certain point. Rolling and hammering the metal when hot have the effect of elongating the crystals into fibres and so increasing the tensile strength, but the iron may be seriously weakened by overheating it. Wrought iron having a fibrous structure has a greater strength measured in the direction of the fibres than it has in a direction at right angles to the fibres.

The ultimate tensile stress of wrought iron may be taken to be about 22 tons per square inch as an average, but it varies with different qualities from about 18 to 26 tons. As a general rule, wrought iron which possesses a very high tensile strength is frequently inclined to be somewhat hard and brittle.

The ultimate strength of wrought iron in compression is rather less than its ultimate tensile stress; it varies from 16 to 20 tons per square inch.

In shear, wrought iron is not quite so strong as in tension; the shearing strength is commonly taken to be seven-eighths of the tensile strength.

21.—**Commercial Tests for Wrought Iron.** It has been mentioned under the heading "Strength of Wrought Iron," that those varieties of wrought iron which possess a very high tensile strength are frequently inclined to be hard and brittle. An iron which was intended to withstand shocks, even if it possessed a high tensile strength, would clearly be quite unsuitable for its purpose if it were at all inclined to be hard and brittle. Consequently, when selecting wrought iron which is to withstand live loads, it is not sufficient to know merely its tensile strength; we must know also to what extent it is ductile.

It is therefore customary, in addition to determining the tensile strength of wrought iron, to determine at the same time how much the metal elongates before breaking, and also in some cases how much it contracts at the part where the breakage occurs.

Thus, in all big contracts for iron and steel work, it is specified that the material must have a tensile strength of not less than a certain figure, and that it shall elongate a certain percentage measured on a certain length. In some instances the percentage reduction of sectional area is also specified, but as the determination of the elongation forms a simple and reliable indication as to the ductility of the material, the contraction of area is commonly not asked for.

The percentage elongation naturally depends on the length of the test piece, and will be greater the shorter the length of the piece. For this reason the length on which the extension is measured must be specified. Usually this length is either 8 or 10 inches.

The best qualities of wrought iron elongate as much as 30 per cent. in a length of 10 inches, whilst inferior qualities may elongate only 5 per cent. or even less. The contraction of sectional area for the former is frequently as much as 50 per cent., whilst for the poor qualities it may only be 10 per cent. or even less.

In addition to tensile and elongation tests, additional tests of wrought iron are sometimes made with the object of ascertaining how the iron will behave if it should require to be forged and worked. Such tests, which are known as *forge tests*, consist principally of bending specimens of the iron, hot or cold, through a certain angle, and noting if the material exhibits any signs of cracking. Thus, wrought-iron rivets, which are sometimes used for boiler and bridge work, although the structures themselves are mostly of mild steel, are required to double when cold without exhibiting any signs of fracture.

Similarly, angle irons may be tested by being bent or flattened out whilst hot.

The quality of wrought iron may also be judged to a large extent by the appearance of the fracture of a broken specimen. A good quality of iron will, generally speaking, show a fibrous fracture, whilst an inferior quality will appear crystalline or laminated, the laminations being due to the presence of slag in thin layers, the slag not having been removed properly during the process of manufacture. It must, however, be pointed out that a crystalline fracture may result through the specimen being subjected to shock, so it does not follow that because a specimen shows a crystalline fracture it must be of inferior quality.

22.—**Steel.** There are a great many varieties of steel used by the engineer, the different varieties depending principally on the percentage of carbon contained.

The percentage of carbon varies from about ·1 to 1·6, so that as cast iron contains from 2 to 5 per cent. of carbon and wrought iron usually not more than ·15 per cent., steel lies intermediate between cast iron and wrought iron.

As might be expected, steel may be made by adding carbon to wrought iron, or by removing a portion of the carbon contained in cast iron. As regards the latter method, the puddling process used for making wrought iron from cast iron by removing the bulk of the carbon from the cast iron can be adopted for making steel by arresting the process before the decarburisation is complete. Steel made by this process is termed "puddled" steel.

The steel used by engineers for machine and structural work is mostly made by one or the other of two important processes, or by a modification of these processes. These are the *Bessemer* and the *Siemens* processes.

23.—In the **Bessemer process,** the whole of the carbon is removed from cast iron so as to leave wrought iron, after which sufficient carbon is added to convert the wrought iron into steel. The process consists of first melting down in a special pear-shaped iron vessel lined with ganister and known as a Bessemer "converter," a quantity of cast iron, and then blowing air through the molten metal. The effect of this is to remove all the carbon from the iron, leaving molten wrought iron. In order to add to the wrought iron the carbon necessary to convert the metal into steel a compound of iron, rich in carbon and manganese, and known as "spiegeleisen," is now added to the molten mass. This completes the process, and a few minutes after the spiegeleisen has been added the converter is turned over and the steel run out into ladles, and thence into iron moulds to form ingots.

Two kinds of steel are made by the Bessemer (and the Siemens) process, viz. "acid" steel and "basic" steel. The acid steel is produced when the converter is provided with an acid lining, e.g. ganister, and the basic steel when the lining is basic in character. In the former case, iron containing phosphorus cannot be treated, but when the converter is provided with a basic lining, not only phosphorus but other impurities may be removed from the iron.

In the Siemens process of steel making, pig iron is melted in a regenerative furnace, after which pure oxidised ores are added from time to time with the object of removing the carbon and silicon in the pig iron. To convert the iron into steel, spiegeleisen and ferro-manganese are then added to the molten metal.

24.—The **Siemens process** of steel making occupies a much longer time than the Bessemer, and on this account is under better control. Samples of the metal produced can be taken from the furnace from time to time and the quality of the steel determined whilst the process is being carried on. For this reason Siemens steel is usually regarded as being more reliable than Bessemer steel.

An important modification of the Siemens process of steel making is that known as the *Siemens-Martin*. In this process a large quantity of scrap wrought iron or steel is melted with the pig iron, so that the percentage of carbon to be removed from the pig iron is less than in the Siemens process. No ore needs be added to effect decarburisation. When the carbon has been sufficiently removed, spiegeleisen or ferro-manganese is added in sufficient quantity to produce steel of the composition required.

On the completion of the Bessemer and Siemens processes, the molten steel is run out into ladles and cast into ingot moulds. When the metal has solidified the ingots are placed in "soaking pits," where they are kept hot for some time. They are then removed from the pits as required and passed to the rolling mills, where they are rolled down, stage by stage, into rails, girders, plates, etc.

The steel produced by the Bessemer and Siemens processes possesses in a high degree the properties of ductility and strength, whilst it is very uniform in character. Such steel is known as *mild* *steel*, and it is used very largely in the construction of all classes of machinery and in the building of ships, bridges, and so on. Mild steel has indeed largely displaced wrought iron, as it possesses almost all the useful properties of the latter metal, whilst it is of even greater strength.

An advantage of the greatest importance possessed by steel is that it can if required be cast into moulds and any desired design of casting thus obtained. Steel castings are greatly superior to those of iron both as regards ductility and strength, and they are consequently of great value to the engineer in cases where more or less complicated designs to resist heavy or variable loads are required. For instance, toothed wheels intended to work under severe conditions are liable to fracture at the teeth if made of cast iron, and are therefore largely made of cast steel in modern practice. Steam pipes intended to carry high-pressure or superheated steam are almost invariably made of cast steel (or wrought steel) in modern practice in preference to cast iron, and in many other connections cast steel is superseding cast iron.

One disadvantage possessed by cast steel is that the metal when cast is often not sound, as it contains numerous " blow-holes " or small cavities. With the object of remedying this the ingots, after being run into moulds of special construction, may be subjected to great pressure in a hydraulic press, the effect of which is to compress the metal and so close up the cavities, at the same time making the steel denser and of greater strength. Steel subjected to this process is known as *Whitworth's compressed steel*, after Sir Joseph Whitworth.

25.—Another important process of steel making is that known as the *cementation* process, which is employed principally for producing steels of hard temper, especially suited for cutting instruments of all kinds.

In this process the steel is produced from wrought iron by adding carbon to the iron. This is effected by placing bars of the purest wrought iron in a converting furnace, covering them with layers of charcoal, and then exposing them to a high temperature for a prolonged period, a week or more, the length of time being governed by the particular quality of the steel required. When the process is completed and the bars taken out of the furnace, the surface of the bars is found to be covered with blisters; for this reason the steel is termed *blister steel*.

Blister steel is inferior in quality, being brittle and irregular in composition, and it must therefore be subjected to further treatment. The bars are consequently cut up into short lengths, piled into faggots, reheated and welded, and then hammered or rolled. The resulting steel is termed *shear steel*. If the latter treatment be repeated, a better quality of steel known as *double shear steel* is produced.

With the object of producing steel of uniform character and composition, blister steel may be melted down in crucibles, run into ingot moulds, and finally worked into plates, bars, etc. The resulting metal is very homogeneous and exceptionally strong, and is known as *crucible cast steel.*

The method usually employed nowadays for producing such steel, however, consists in making the metal direct from the best wrought-iron bars, the bars being cut up and placed in crucibles, small quantities of charcoal, and subsequently spiegeleisen, or ferro-manganese, being added to carburise the metal to the extent required.

Crucible steel, in addition to being of great strength, may be made extremely hard by being heated to redness and then plunged in cold water, and it is therefore particularly suited to the manufacture of cutlery and cutting tools.

The steel made by the cementation process is commonly termed *hard* steel, to distinguish it from mild steel as made by the Bessemer and Siemens processes; the former usually contains over ·75 per cent. of carbon, and the latter less than ·5 per cent.

26.—**Alloy Steels.** Of recent years great developments have taken place in the production of special steels known as *alloy steels.* These consist of iron and carbon with a quantity of some other element added, such as nickel, chromium, etc. The addition of these elements gives to the steel certain properties which render it especially suitable for some particular purpose. *Nickel steel,* for example, is much stronger and possesses a much higher elastic limit than ordinary mild steel, whilst it is better able to resist corrosion than mild steel. For these reasons it is now being largely used in the manufacture of certain parts of engines and motor cars (crank shafts and connecting rods, for instance), and also in the manufacture of boiler tubes. Owing to its high elastic limit, nickel steel is particularly suited to resist shocks, vibration, etc.; hence its adoption in the manufacture of crank shafts, connecting rods, and suchlike. As regards the adoption of nickel steel for boiler tubes, experiments appear to show that nickel steel boiler tubes will last twice as long as mild steel tubes, and in view of this and the fact that its elastic limit and ultimate strength are so much higher, a lighter gauge of tube may be employed with safety, an important advantage in connection with the boilers for torpedo-boat destroyers and the like. *Chromium steel* possesses great tensile strength and hardness, whilst it is more or less malleable, providing the percentage of chromium present is not too high; it can be hardened like hard steel.

27.—**Special Steels for resisting Vibration and Shocks.** We have seen that when a piece of iron or steel is exposed to a large number of repetitions of stress it tends to suffer deterioration in some way, so that it may eventually fail with a stress much less than the true breaking stress.

Where the conditions of working are such that the stresses are sudden and variable, or alternating, it is important that the material used in the manufacture of the different members of the machine or structure should be of special quality and capable of resisting satisfactorily for long periods the stresses imposed upon it. The advent of the motor car, many of the working parts of which are exposed to extremely severe conditions of working, is no doubt largely responsible for the introduction of special steels for resisting repeated and variable stresses. Such steels must possess both a high elastic and a high tensile strength, whilst they must be particularly tough and ductile.

A steel which has been found to possess these qualities in a high degree is that known as *vanadium steel*, which consists of iron and carbon together with a certain percentage of vanadium, and generally a certain amount of chromium. This particular steel, which is manufactured by Willans and Robinson, of Rugby, possesses a high tensile strength, but unlike ordinary steel of high tensile strength it is not brittle, as it contains only a low percentage of carbon.

Another type of steel particularly suited to motor-car construction is *Vickers nickel-chrome steel*, manufactured by the famous firm, Vickers, Sons, and Maxim. This, as its name will imply, is steel containing both nickel and chromium.

From certain tests which have been carried out to show its capability of resisting fatigue, it appears that this steel has ten times the strength of ordinary mild steel, i.e. it will withstand ten times as many applications of the load without failure as will mild steel.

28.—**Strength of Steel.** As there are many distinct varieties of steel, the strength of this metal naturally varies within fairly wide limits.

Mild steel, which is the class of steel of most concern to the machine designer, has a tensile strength of from 27 tons to 33 tons per square inch.

The tensile strength of the steel used in the manufacture of crank shafts, piston rods, etc., frequently has a tensile strength of about 30 tons per square inch.

Mild steel is almost universally employed in the construction of boilers and bridges. At one time wrought iron was mostly used in boiler work, but this metal has been superseded by mild steel, which is generally more ductile, and much stronger in tension.

For bridge construction mild steel having a tensile strength of 30 tons per square inch is commonly employed. Thus the steel used in the construction of the tension members of the great Forth Bridge in Scotland had a tensile strength of 30 to 33 tons per square inch.

The strength of Whitworth's fluid compressed steel is considerably greater than that of ordinary mild steel. This steel may indeed be manufactured to have an ultimate stress of anything from 40 to 70 tons per square inch, depending on the particular purpose for which it is required.

Some of the varieties of cast steel possess a very high tensile strength, as much as from 50 to 60 or even 70 tons per square inch, although some varieties have a strength very much below this figure. Certain of the alloy and motor-car steels also possess a high tensile strength. One of the strongest steels made is that employed in the manufacture of wire, such as is used for wire ropes, etc. Such steel has frequently a strength of 90 tons per square inch, whilst in some cases the strength is considerably greater even than this.

As regards the compressive strength of steel, this also varies greatly with different varieties.

Thus the steel used for the compression members of the Forth Bridge had an ultimate compressive stress of 34 to 37 tons per square inch, whilst some hardened steels which contain a large amount of carbon have an ultimate stress of nearly five times this.

The shearing strength of all classes of steel is generally lower than the tensile strength, the ratio varying from about two-thirds for Bessemer and crucible steel to nearly seven-eighths for Siemens-Martin steel.

29.—**Commercial Tests for Steel.** The remarks made under the heading "Commercial Tests for Wrought Iron" apply in large measure to steel, particularly to mild steel. Thus, in addition to having a certain tensile strength, mild steel must show a certain elongation and contraction of area, and must be capable of being bent through a certain angle or to a curve of a certain radius without exhibiting signs of fracture.

Steel used by the Admiralty for shipbuilding is required to have an ultimate tensile strength of not less than 26 tons per square inch and not more than 30, with an elongation of 20 per cent. in a length of 8 inches. Strips must be cut from the plates, heated uniformly to a low cherry red, and cooled in water at a temperature of 80 degrees F., and then must stand bending in a press to a curve of which the inner radius is one and a half times the thickness of the steel tested.

The Board of Trade require the steel used for the shell plates of steam boilers to have an ultimate tensile strength of 27 to 32 tons per square inch, with an elongation of not less than 20 per cent. on a length of 8 inches.

Furnace plates exposed to flame must have an ultimate tensile strength of 26 to 30 tons per square inch with not less than 23 per cent. elongation on 8 inches. A shearing from each plate exposed to flame is taken and subjected to the same test as that required by the Admiralty. The test for plates not exposed to flame consists in bending the test piece cold to a curve, the inner radius of which is equal to one and a half times the thickness of the plate.

The appearance of the fractured surface of steel gives some indication as to the quality of the steel, but only the carrying out of tensile and other tests such as above described furnish the information necessary to prove that the steel is of the desired quality for the purpose for which it is intended.

30.—**Copper.** This metal, like iron, is extracted from ores, the metal is not of sufficient importance to the machine designer but to call for a description of the methods by which it is extracted.

The special properties possessed by copper are toughness, malleability, and ductility. The pure metal may be worked up by hammering, rolling, and wire-drawing into a state of great strength, but these operations tend to make it more or less hard and brittle.

Owing to its malleability and ductility copper may be readily hammered into various shapes, rolled into plates, or drawn into wires. It is mostly used by the mechanical engineer for the manufacture of steam pipes, locomotive fire boxes, and for bolts used under such conditions as would cause them to corrode if made of iron or steel, but as it is an exceptionally good conductor of electricity it is largely used by the electrical engineer for many purposes.

Copper pipes are mostly made from sheets which are bent cold, the joints being brazed, but such pipes may be made by electro-deposition. Small pipes are usually made by drawing, these being known as *solid drawn* pipes.

The average tensile strength of copper, as cast from the ore and refined, is about 10 tons per square inch, but it varies from 9 to 12 tons. By rolling, the strength may be increased to 15 or 16 tons per square inch, and by wire-drawing to as much as 30 tons. Copper can thus be manufactured of the strength almost of wrought iron or mild steel.

31.—**Brass.** Brass is an alloy of copper and zinc, or an alloy of copper and zinc with a little lead and tin. A common quality consists of two parts of copper by weight to one of zinc. Lead is sometimes added in small quantities with the object of making the metal softer and more easy to machine. The addition of tin tends to cause the metal to break up more readily under the action of a cutting tool. The tensile strength of brass varies according to the composition; it may be as little as 7 tons per square inch or as much as 13 tons, an average being about 9 tons. The compressive strength of brass is about 5 tons per square inch.

Brass is much employed in the manufacture of smoke tubes for locomotive type boilers, and for condenser tubes; it does not corrode when exposed to the action of water to the same extent as does iron or steel.

32.—**Muntz Metal** is a form of brass, and differs from the latter metal principally on account of the greater proportion of zinc it contains; it has a much higher tensile strength than brass, viz. about 22 tons per square inch.

As it does not rust or become affected to any appreciable extent when exposed to the action of sea water, it is suitable for the manufacture of bolts exposed to conditions which would cause them to rust if made of iron or steel.

33.—**Bronze** or **Gun Metal** is an alloy of copper and tin, the former usually being present in much the larger proportion. The greater the proportion of the tin present, the harder is the metal. The hard varieties of bronze are much used for shaft bearings.

Various varieties of bronze are obtained by introducing small percentages of certain elements. Thus, the introduction of phosphorus with the elements composing the alloy gives us *phosphor bronze,* which is generally harder and stronger than ordinary bronze.

The properties of phosphor bronze may be varied as desired, and the alloy is employed in the manufacture of boiler fittings, shaft bearings, tubes, pump rods, and for numerous other purposes.

34.—**Manganese Bronze** is obtained by adding ferro-manganese to bronze. This alloy is largely employed for the propeller blades of steamships, as it possesses the two qualities required of such blades, viz. great strength, and power to resist the action of sea water.

35.—**Delta Metal** is another variety of bronze which can be made tough and hard. It is very suitable for making sound fine castings, and is extensively used in hydraulic work on account of its anti-corrosive properties.

36.—**Babbit's White Metal** is an alloy of tin, antimony, and copper. It is a white metal especially suited for lining the bearings of shafts, as it wears smoothly and reduces friction losses.

37.—**Aluminium.** Some reference should be made to this metal, which possesses remarkable properties. Aluminium is a white-coloured metal of exceptional lightness, being only about one-third the weight, bulk for bulk, of iron. It is very malleable and ductile, and it does not rust if pure.

It has a tensile strength of from 6 to 8 tons per square inch as cast, but as it may be drawn into wire and hammered, its strength may be greatly increased.

Aluminium may be alloyed with other elements, such as copper and silicon, to form what are known as aluminium bronzes, some of which possess high tensile strength and great toughness. Where both lightness and strength are desired, a mixture of aluminium and a small proportion of copper forms a most suitable alloy.

38.—**Timber.** At one time timber was employed to a fairly large extent in constructional work, but it has since been almost entirely superseded by iron and steel. For this reason it is unnecessary to do more than merely refer to the strength of timber and to the use of one or two woods still employed by the engineer for special purposes.

Timber is frequently used in temporary structures, and may be required to sustain fairly heavy loads. In view of this a designer should have some knowledge of the strength of timber in tension and compression.

English oak has a tensile strength varying from 3 to 9 tons per square inch, teak 2 to 7, pitch pine 2 to 5, beech and ash both 2 to 7, and elm 3 to 7. The average strength of timber is thus seen to be very variable, but a mean figure of 4 tons per square inch is often taken, when the particular kind of wood concerned is not known with certainty.

The strength of timber to resist crushing varies from 2 to 5 tons per square inch, and a mean of 3½ tons per square inch may be adopted.

It is generally known that timber after being in use a certain length of time tends to rot. The engineer, therefore, who is called upon to determine whether or not an existing timber beam is sufficiently strong to carry a certain load should adopt a large factor of safety to make allowances for deterioration of the strength of the wood and for the uncertainty of the particular kind and quality of the wood. The modulus of elasticity of timber varies from about 1,000,000 to 3,000,000 lbs. per square inch.

The principal uses to which timber is put at the present time by the machine designer are for the construction of pulleys, the sheaves of pulley blocks, and for shaft bearings. It may be mentioned that wood pulleys are sometimes employed for belt driving, as they possess certain advantages over iron pulleys.

Oak and boxwood are occasionally used for bearings of machinery. For bearings which have to work in water, a special wood known as *lignum-vitae* is very suitable and is much used in such cases.

39.—**Masonry.** Other materials used by the engineer in addition to those mentioned in the foregoing are *brickwork, stone-work,* and *concrete.* These materials are used in the building of machinery and engine foundations, piers, and so on, and are generally employed for sustaining compressive loads only.

The crushing strength of brickwork as a whole is much less than that of the individual bricks, which have an ultimate strength varying from $\frac{3}{4}$ ton to 3 or 4 tons per square inch. The mortar is an important factor in determining the actual strength of the brickwork.

The crushing strength of stone varies considerably, but an ultimate stress of 3 tons per square inch may be taken as an average figure.

The crushing strength of concrete averages about 1 ton per square inch.

Of recent years great developments in the uses of concrete for constructional purposes have taken place. As used by itself concrete is of little use to the engineer for taking tensile loads, because its tensile strength is very low. It is, however, practicable to reinforce the strength of the concrete by embedding in it steel bars, matters being so arranged that the steel work takes the tensile stresses, and the concrete the compressive stresses. Concrete reinforced in this way is known as *reinforced concrete,* and in the construction of buildings, beams and columns of reinforced concrete are being very largely used at the present time.

Chapter III

CALCULATION OF SIMPLE STRESSES

40.—HAVING read the previous descriptive matter carefully, the student should now have clear ideas regarding stresses and strains and the characteristic properties of engineering materials, and we shall next consider how such stresses and strains are calculated, with the object of determining whether or not any particular piece of machinery is capable of withstanding safely the load which may be imposed upon it under actual working conditions.

In the case of a simple tensile or compressive force acting on a bar or column, we have the following relation:—

Let F=total force or load in pounds,
,, A=area of section in square inches,
,, f=stress in pounds per square inch.

Then $f=\frac{F}{A}$.

If the total force F be given in tons instead of pounds, the same relation still holds good, but the stress f will then be in tons per square inch instead of pounds. Similarly, if the force F be in tons, and the area of the section in square feet, the stress f will be in tons per square foot.

This formula applies equally to bars or rods in pure tension, or to columns or struts in pure compression.

There is no better way of obtaining a sound knowledge of the strength of materials than working through many examples. A student will indeed learn more from a careful study of one or two examples than from a perusal of many chapters of subject-matter.

A number of examples, with complete solutions, will be given, therefore, as a guide, and the student should, as far as possible, follow the methods indicated in working through any examples which may come before him. In giving the answers to the questions, it is to be noted that there is no necessity whatever to work out the results correct to six or seven significant figures, three or four being quite sufficient for general purposes. It is, for instance, absurd to say that the stress in a body is 30·50845 tons per square inch, when we consider how many factors may affect the correctness of the calculated result. If we say the stress is 30·5 or 30·51 tons per square inch, this is quite near enough for all practical purposes. The slide rule gives results which are correct to three significant figures, and may be used with great advantage and saving of time in calculating the results.

WORKED EXAMPLES

41.—(1) A round steel bar, 2 inches diameter, is subjected to a total pulling force of 7500 lbs. What is the resultant stress in the material?

NOTE.—It is always advisable when working out problems of this nature, or in fact problems generally, by means of formulæ, to write down first the particular formula applicable to the problem, and also to state exactly what each letter of the formula represents. Many students, when solving problems, simply write down a mass of figures, which may be intelligible to themselves, but which cannot possibly be followed by any one else. This method of working is not only slovenly, but it is very objectionable for two important reasons. In the first place, the student is extremely liable to make mistakes, as he loses sight of the problem after a time if the latter be at all long; and in the second place, it is quite impossible for an examiner to correct the work and point out any mistake if the various steps in the working be not clearly indicated. Generally speaking, time will be gained rather than lost if each step in the working be made as clear as it is possible to make it.

Solution.—The relation between the force or load and the stress acting on a bar is given by the formula,

$$f=\frac{F}{A},$$

where f=stress in pounds per square inch,
F=total force or load in pounds,
A=area of section of bar in square inches.

In the question, the data given are the force acting, F, and the diameter of the bar. The area A can of course be found when we know the diameter: it is equal to $\frac{\pi}{4}d^2$ or πr^2, where d=the diameter and r=the radius of the bar in inches.

We have then

$$f=\frac{F}{A}=\frac{F}{\frac{\pi}{4}d^2}=\frac{F}{\cdot785d^2}.$$

F=7500 lbs.
d=2 inches.

$$\therefore f=\frac{7500}{\cdot785\times2^2}=\frac{7500}{3\cdot14}=2390 \text{ lbs. per square inch.}$$

$\therefore$ Resultant stress in material=2390 lbs. per square inch.

(2) What is the stress on a boiler stay $1\frac{1}{4}$ inches diameter at the least section, if the steam pressure in the boiler is 25 lbs. per square inch and the stays are spaced 18 inches apart? (Mar. Eng. Second-Class Exam.)

A little explanation may be necessary in connection with this problem. The object of the stays in a boiler is to stiffen such flat surfaces as there may be which are liable to bulging or bursting as a result of the pressure acting upon them. In the case of a marine boiler, the front and back end plates are stayed by means of a number of long bolts or stays which pass through the two plates and which are nutted at each end. (See Fig. 7.) The question does not state how many stays there are, or the area of the plates stayed, but this is not necessary, because the distance apart of the stays is given. Referring to Fig. 7 (a), which shows an elevation of a portion of the stayed plates, it will be understood that each stay will support the area represented by the shaded square, which area is $18\times18=324$ square inches. As there is a pressure of 25 lbs. on each square inch of the surface, the total pressure acting on each stay will be $25\times324=8100$ lbs.

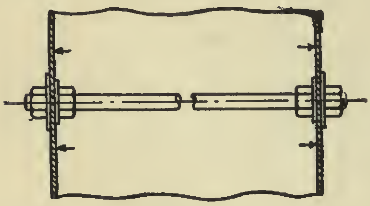

Fig. 7

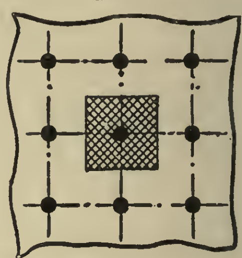

Fig. 7(a)

Using the relation, $f=\frac{F}{A}$, where f=stress in stay,

F=total force on stay,
A=sectional area,

we have

$$f=\frac{8100}{\frac{\pi}{4}\times1{\cdot}25^2}=\frac{8100}{{\cdot}785\times1{\cdot}562}=6610 \text{ lbs. per square inch.}$$

$\therefore$ Stress in each stay=6610 lbs. per square inch.

NOTE.—It should be observed that, strictly speaking, the area on which pressure acts per stay is slightly less than $18\times18=324$ square inches, because no pressure acts at that part of the plate through which the stay passes. The area of this part is of course the area of the section of the stay. As an exercise, the student may calculate what the stress is when this is taken into account. He will find the difference is very slight.

(3) A round rod of wrought iron is required to sustain a tensile load of 20 tons. If the ultimate tensile stress of wrought iron be taken to be 22 tons per square inch, and a factor of safety of five be required, what must be the diameter of the bar?

In this question, the first thing to be determined is the safe working stress, f.

Now the safe working stress is obtained by dividing the ultimate stress by the factor of safety; thus,

$$\text{Safe stress} = \frac{\text{ultimate stress}}{\text{factor of safety}} = \frac{22}{5} = 4{\cdot}4 \text{ tons per square inch.}$$

We have now to find what diameter of bar is required to sustain a load of 20 tons if each square inch of section is to carry 4·4 tons. It will be best to find the area of the section first, and this is found from the relation $f=\frac{F}{A}$, where the letters represent the factors already specified.

As we require to find A in this case, F and f being already known, it is convenient to rearrange the formula, or to state what the factor to be found, A, is. Thus

$$A = \frac{F}{f} = \frac{20}{4{\cdot}4} = 4{\cdot}55 \text{ square inches.}$$

Thus the rod must have a sectional area of 4·55 square inches. To find the diameter from the sectional area, we use the relation $\frac{\pi}{4}d^2 = A$, where d=diameter. From this,

$$d^2 = \frac{A}{\frac{\pi}{4}} = \frac{4A}{\pi} \quad \therefore\ d = \sqrt{\frac{4A}{\pi}} = 2\sqrt{\frac{A}{\pi}}.$$

We have just found A to be 4·55 square inches.

$$\therefore\ d = 2\sqrt{\frac{4{\cdot}55}{\pi}} = 2\sqrt{1{\cdot}45} = 2 \times 1{\cdot}203 = \underline{2{\cdot}406 \text{ inches.}}$$

A bar 2·406 inches diameter would therefore be required. In actual practice, a bar $2\frac{1}{2}$ inches diameter would most likely be used, as it is not necessary to make the diameter of exactly the size calculated, but to take the nearest one-sixteenth or one-eighth of an inch, keeping of course on the safe side.

42.—**Calculations of Simple Strains.** In Chapter I. it was explained that a tensile strain is, strictly speaking, a ratio, i.e. the ratio of the increase of length of a body, due to the stress, to the original length of the body.

Taking the simple case of a bar subjected to a tensile load,

let l=length of the bar before applying the load,
x=extension due to the load,
e=strain.

Then the general relation is,

$$e=\frac{x}{l}.$$

It is to be noted that x and l may be measured in any units of length, such as inches or feet, because the strain $\frac{x}{l}$ is simply a ratio, and therefore independent of the units employed. It is, however, important to note that whatever unit of length be employed, the same unit must be used for both x and l. For instance, if l be measured in inches, the extension x must also be measured in inches; whilst if l be measured in feet, x must likewise be measured in feet. So long as the units are the same, the ratio is not altered.

Worked Examples

43.—(1) A steel wire 8 feet long is fixed at its upper end and is extended $\frac{1}{4}$ inch by a load applied to its lower end. What is the strain produced?

The general relation for tensile strain is,

$$e=\frac{x}{l}$$

where e=the strain, x=the extension, and l=the original length of the wire. The extension x is given as $\frac{1}{4}$ inch and the length as 8 feet. The unit of length adopted must be the same for the extension and the length, so taking the inch as the unit, x will be ·25 and l will be $8\times12=96$. Hence, $e=\frac{\cdot25}{96}=\cdot0026$. The strain is therefore ·0026.

Note.—A common mistake on the part of students is to write the strain as a fraction of an inch. It must not be forgotten that the answer is merely a fraction, and nothing must be written after it.

(2) A cast-iron column supporting a heavy load is shortened $\frac{1}{16}$ inch by the load. If the original length of the column was 12 feet, what is the strain?

This question is similar to the previous one, except that the strain is compressive instead of tensile, the column being compressed instead of extended.

We have
$$e=\frac{x}{l}$$
where e=the strain, x=the amount of shortening of the column, and l=the original length of the column.

$$x=\tfrac{1}{16}\text{ inch}=\cdot 0625\text{ inch},\ l=12\times 12=144\text{ inches.}$$

$$\therefore\ e=\frac{\cdot 0625}{144}=\cdot 000434.\quad \text{The strain then is } \underline{\cdot 000434.}$$

44.—**Calculations of Moduli of Elasticity.** The modulus of elasticity of a material, as pointed out previously, although an imaginary constant, plays an important part in engineering calculations, as shall now be seen. We may remind the student that the constant is obtained by dividing the stress in a body by the strain which results from the stress, and that it represents, for any particular material, the load which would double the length of a bar of that material, one square inch sectional area, providing the bar remained perfectly elastic and did not fail whilst the load was being applied.

Let F=total tensile force or load in pounds acting on a bar.
,, A=sectional area of bar in square inches.
,, f=stress in bar in pounds per square inch.
,, l=original length of bar in inches.
,, x=extension of bar due to the load in inches.
,, e=the strain.
,, E=modulus of elasticity.

According to the definition, Modulus of elasticity$=\dfrac{\text{stress}}{\text{strain}}$.

Using the above symbols, $E=\dfrac{f}{e}$ from which $f=Ee$. Now we have seen that $f=\dfrac{F}{A}$ and $e=\dfrac{x}{l}$.

Substituting these relations in the equation $E=\dfrac{f}{e}$, we have

$$E=\frac{f}{e}=\frac{\frac{F}{A}}{\frac{x}{l}}=\frac{Fl}{Ax}.$$

This may be written, $\underline{EAx=Fl.}$

Worked Examples

45.—(1) In a tensile test of a steel wire, a load of 100 lbs. is found to produce an extension of ·039 inch. If the diameter of the wire is $\frac{1}{8}$ inch and the length 12 feet, find the stress, the strain, and the modulus of elasticity of the steel.

Let F=force applied to wire=100 lbs.
,, A=sectional area of wire in square inches.
,, d=diameter of wire in inches.
,, f=stress in lbs. per square inch.

Then $f=\frac{F}{A}=\frac{F}{\cdot 785d^2}=\frac{100}{\cdot 785\times\cdot 125^2}=8150$ lbs. per square inch.

Stress=8150 lbs. per square inch.

Let e=strain.
,, x=increase of length of wire in inches.
,, l=original length of wire in inches.

Then $$e=\frac{x}{l}=\frac{\cdot 039}{12\times 12}=\cdot 000271.$$

Modulus of elasticity$=\frac{\text{stress}}{\text{strain}}=\frac{8150}{\cdot 000271}$
=30,100,000 lbs. per square inch.

(2) Taking the modulus of elasticity of wrought iron to be 29,000,000 lbs. per square inch, find how much a wrought iron rod 18 feet long and 1 square inch sectional area would be extended by a steady load of 10 tons. What would be the extension if the rod was 2 square inches sectional area instead of 1?

We are given that the modulus of elasticity of wrought iron is 29,000,000 lbs. per square inch, which means to say that a rod of wrought iron 1 square inch in sectional area and of any length, would be doubled in length by a force of 29,000,000 lbs. The question is therefore one of simple proportion thus :—

If a force of 29,000,000 lbs. will produce an extension of 18 feet, what extension will a force of 10 tons produce?

18 feet $=18\times 12=216$ inches.
10 tons$=10\times 2240=22,400$ lbs.

Then 29,000,000 : 216 :: 22,400 : x, from which $x=\frac{216\times 22,400}{29,000,000}$
$=\cdot 167$ inch.

The rod would be extended by $\cdot 167$ inch, or roughly $\frac{1}{6}$ of an inch.

The answer to the second part of the question is easily obtained if we remember one of the laws which govern the extension of a bar subjected to tensile loads, viz. the amount of extension is inversely proportional to the sectional area of the bar.

If the sectional area then be doubled, the amount of the extension will be halved, and one half of $\cdot 167=\cdot 0835$ inch.

Alternative Method.—In case the above method of working is not quite clear, the following method may be adopted. First find the stress in the rod; then from the relation,

$$\text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}},$$

find the strain, the modulus being known.

Having found the strain and knowing the length of the rod, the extension may be found from the relation,

$$\text{Strain} = \frac{\text{increase of length}}{\text{original length}}.$$

Thus, $\text{Stress} = \dfrac{\text{total load}}{\text{sectional area}} = \dfrac{10}{1} = 10$ tons per square inch.

The modulus is given in lbs. per square inch, so the stress should also be given in lbs. per square inch.

Stress $= 10 \times 2240 = 22{,}400$ lbs. per square inch.

Modulus of elasticity $= \dfrac{\text{stress}}{\text{strain}}$, from which strain $= \dfrac{\text{stress}}{\text{modulus}}$.

Substituting the known values, strain $= \dfrac{22{,}400}{29{,}000{,}000} = \cdot 000773$.

$$\text{Strain} = \frac{\text{increase of length}}{\text{original length}}, \text{ from which}$$

Increase of length $=$ strain $\times$ original length.

$= \cdot 000773 \times 18$ (feet)

$= \cdot 000773 \times 18 \times 12$ (inches) $= \cdot 167$ inch.

Further Alternative Method.—The problem may also be worked by the use of the general relation already given, viz.

$$EAx = Fl,$$

where F = total tensile force on bar.
l = original length of bar.
E = modulus of elasticity.
A = sectional area of bar.
x = extension due to the load.

What we require to find is the extension, x.

Rearranging the equation, we have

$$x = \frac{Fl}{EA}.$$

$F = 10 \times 2240 = 22{,}400$ lbs
$l = 18 \times 12 = 216$ inches.
$E = 29{,}000{,}000$ lbs.
$A = 1$ square inch.

$$\therefore\ x = \frac{22{,}400 \times 216}{29{,}000{,}000 \times 1} = \underline{\cdot 167 \text{ inch.}}$$

(3) A steel piston rod is 8 inches diameter, and the diameter of the engine cylinder is 88 inches. Considering only the stroke when the piston rod is in compression, find how much the rod is shortened and also the stress in the rod, the length of the latter being 9 feet and the effective pressure 40 lbs. per square inch. Assume the modulus of elasticity of the steel from which the rod has been made to be 29,000,000 lbs. per square inch.

First find the total load compressing the rod, i.e. the total load on the piston, which is equal to the area of the piston multiplied by the effective pressure.

Total load=area of piston×effective pressure.

$$=\frac{\pi}{4}\times 88^2\times 40=243{,}300 \text{ lbs.}$$

If the rod were 1 square inch in sectional area, we assume that a load of 29,000,000 lbs. would shorten it to the extent of $9\times 12=108$ inches.

Still making this assumption, what amount of shortening would be produced by a load of 243,300 lbs. ?

Stating this as a proportion sum, we have

$$29{,}000{,}000 : 108 :: 243{,}300 : x$$

$$x=\frac{108\times 243{,}300}{29{,}000{,}000}=\cdot 905 \text{ inch.}$$

The rod would thus be shortened ·905 inch if it were of 1 square inch sectional area. Actually the sectional area of the rod is $\frac{\pi}{4}\times 8^2=50\cdot 26$ square inches. The extension or shortening is inversely proportional to the sectional area, so if the area is 50·26 square inches instead of 1 square inch, the actual shortening will be $\cdot 905\div 50\cdot 26=\cdot 018$ inch.

During one stroke, then, the piston will be shortened by ·018 inch, or nearly one-fiftieth of an inch.

$$\text{Stress in rod}=\frac{\text{total load}}{\text{sectional area}}=\frac{243{,}300}{50\cdot 26}=\underline{4840 \text{ lbs. per square inch.}}$$

During the remaining stroke, it may be mentioned, the rod will be in tension, when it will be extended, the amount of the extension being the same as the amount of shortening during the previous stroke, assuming the same conditions throughout. The total alteration of length of the piston rod during each revolution would consequently be twice ·018 inch, i.e. ·036 inch. We are here neglecting the small piston rod area, which should, strictly speaking, be deducted from the area of the piston on one side.

Chapter IV

FURTHER CONSIDERATION OF STRESS AND STRAIN—COMPOUND AND REPEATED STRESSES

46.—In the previous chapters, only stresses of a very simple character, mostly simple tensile and compressive, have been dealt with, and a little attention must now be devoted to the consideration of stresses which are not quite so simple.

47.—**Normal and Tangential Stresses.** When we say that a bar or rod is exposed to a simple tensile or compressive stress of so many pounds or tons per square inch of sectional area, we generally assume the section to be in a plane at right angles to the axis of the bar, the stress acting normally to the section. Now, in general, a stress may act in a direction at any angle with any section in a loaded bar or other body; thus it may act either normally to the section, obliquely, or tangentially, i.e. in a direction parallel to the section.

When the stress acts normally, the tendency is for the two portions lying on the two sides of the section to be drawn directly away from each other, or to be closed directly against each other, depending, of course, on whether the stress is tensile or compressive.

When the stress acts tangentially, the tendency is for the portion on one side of the section to slide over the other portion; in other words, the stress is a shear stress.

If the direction of the stress is oblique to the section, then it can be shown that the general effect is equivalent to a certain stress acting normally to the section, together with another stress acting tangentially. For instance, suppose XY in Fig. 8 to be an imaginary section of a loaded bar, and suppose a stress f acts obliquely to the section as indicated. Then the stress f may be resolved into normal and tangential components. If θ be the angle between the normal to the section and the line of direction of the stress, then the normal component of the stress is $f \cos \theta$, and the tangential component $f \sin \theta$.

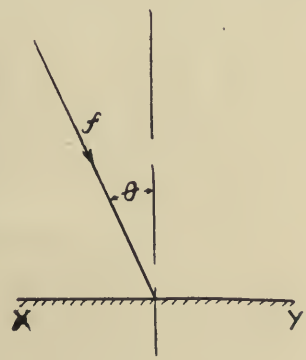

Fig. 8

The stress f acting at an angle θ to the normal to the section has therefore the same effect as a stress equivalent to $f \cos\theta$ acting normally to the section, and a tangential or shearing stress of $f \sin\theta$ acting parallel to the section.

In Fig. 9 is shown a block of metal subjected to a direct compressive load. Considering any horizontal section of the block, if F represent the total load, and A the area of the section, we know from our previous work that the stress f acting over the section is $\frac{F}{A}$. If next we consider any inclined or oblique section, the stress acting over that section, due to the same load, will clearly be different in amount from that acting over any horizontal section. As regards the oblique section, there will be both a normal and a tangential stress acting over the section, whereas the stress on the horizontal section will be purely normal.

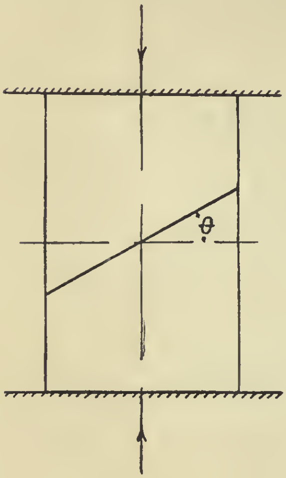

Fig. 9

To find the magnitude of the normal and tangential stresses over the oblique section, we must resolve the load F into normal and tangential loads. Referring to Fig. 9, if we assume the oblique section to make an angle θ with the horizontal section, the normal load on the former will be $F\cos\theta$ and the tangential load $F\sin\theta$.

The area of the oblique section, which for convenience of reference may be represented by A_o, is $\frac{A}{\cos\theta}$.

Let f = the normal stress acting over the horizontal section.
,, F_n = ,, ,, load on the oblique section.
,, f_n = ,, ,, stress acting over the oblique section.
,, F_t = ,, tangential load on the oblique section.
,, f_t = ,, ,, stress acting over the oblique section.

$$\text{Then } f_n = \frac{\text{normal load}}{\text{area of section}} = \frac{F_n}{A_o} = \frac{F\cos\theta}{\frac{A}{\cos\theta}} = \frac{F\cos^2\theta}{A} = \underline{f\cos^2\theta.}$$

$$f_t = \frac{\text{tangential load}}{\text{area of section}} = \frac{F_t}{A_o} = \frac{F\sin\theta}{\frac{A}{\cos\theta}} = \frac{F\sin\theta\cos\theta}{A} = \underline{f\sin\theta\cos\theta.}$$

Now $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$, $\therefore f_t = \underline{\frac{f\sin 2\theta}{2}}$.

The sine of an angle has its maximum value when the angle is 90 degrees, the value then being 1. For the maximum value of the shearing stress, therefore, 2θ must equal 90 degrees, i.e. θ must be 45 degrees.

It follows, then, that when a block of material is subjected to a compressive load, a shearing stress is produced on all oblique sections, the greatest stress being produced on sections making an angle of 45 degrees with the horizontal sections. For this angle,

$$f_t = \frac{f \sin 90^\circ}{2} = \frac{f}{2},$$

or one half the compressive stress acting on the horizontal sections.

The value of f_n for this angle is also $\frac{f}{2}$, i.e. $f\cos^2\theta = f\cos^2 45$ $= f\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{f}{2}.$

The above is, of course, a theoretical consideration, but that it is generally correct is proved by the fact that blocks of brittle metal, masonry, etc., when subjected to compressive loads, commonly fail by shearing along oblique planes, approximately at 45 degrees to the direction of action of the load.

48.—**Shear Stresses.** *A shear stress is always accompanied by a similar stress of equal intensity at right angles to it.*

This is an important theorem, which may be proved in the following way. Let the square, *abcd*, in Fig. 10 represent a cube of unit thickness subjected to shearing stresses f_s acting along the opposite sides *ab* and *cd*. The general tendency of these stresses, which form a couple, is to cause the cube to rotate, and, in order to maintain equilibrium, another couple of equal moment and opposite sense would be required. This couple would be given by stresses acting along the other sides, *ad* and *bc*, in the directions indicated by the arrows. If we let the stresses required to maintain equilibrium be represented by f_x, then by taking moments about any one of the four corners, *a*, *b*, *c*, or *d*, we can find the relation between f_s and f_x. Thus, taking moments about the corner *c*,

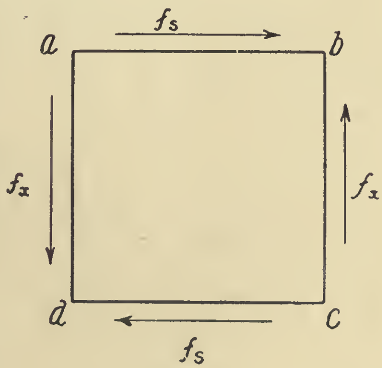

Fig. 10

$$f_s \times ab \times bc = f_x \times ad \times cd.$$

The products, $ab \times bc$ and $ad \times cd$, are equal to each other, so cancelling out we get $f_s = f_x$.

The shearing stress f_s must therefore be accompanied by a shearing stress f_x of equal intensity acting at right angles to it, as otherwise the cube would not remain in equilibrium.

Note that $(f_s \times ab)$ and $(f_x \times ad)$ represent the total forces acting along the sides *ab* and *ad* respectively, the lengths *bc* and *cd* representing the leverages at which these forces act.

A pure shear stress is equivalent to two equal and opposite stresses, one tensile and the other compressive, acting at right angles to each other.

This is another important theorem which we proceed to explain. Reverting to the cube of Fig. 10, we have seen that if a stress f_s act along the opposite sides *ab* and *cd*, there must be a stress of equal intensity acting along the other sides *ad* and *bc* in the direction indicated by the arrows. Assume now the cube to be divided along one of the diagonals into two halves (see Fig. 11), and consider the equilibrium of one of these halves, say *acd*. There are equal stresses acting along the sides *ad* and *cd*, and if the half of the cube under consideration be in equilibrium, there must be a third stress acting normally to the diagonal *ac*. That this is so will be made clearer if we consider that the two forces F_s which give rise to the two stresses f_s have the same effect as a single force or resultant which may be determined both in magnitude and direction by the well-known theorem, the parallelogram of forces. The construction of the parallelogram of forces for the case in point is shown by dotted lines, the diagonal of the parallelogram representing the resultant effect of the two forces F_s. Clearly, to balance this resultant effect, there must be an equal force acting in the opposite direction from the diagonal *ac*. The magnitude of this force is $\sqrt{2}\,F_s$, which, of course, is greater than F_s, but as it acts over a surface which is greater in the proportion of $\sqrt{2}$ to 1 than the surfaces over which F_s acts, the stress or force per unit area is precisely the same as that due to F_s, viz. f_s. The stress acting over *ac* will be a tensile stress.

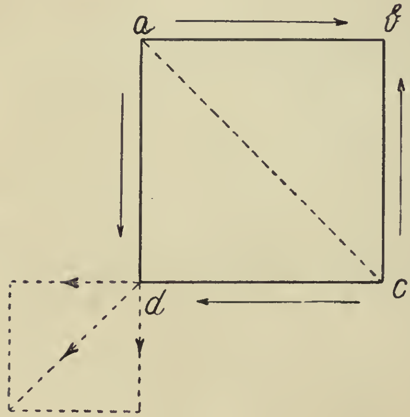

Fig. 11

Exactly the same reasoning may be applied to one of the halves made by cutting the cube along the diagonal *bd*, to show that there is a stress of equal intensity along *bd*. The latter stress will, however, be found to be of the opposite kind to that on the diagonal *ac*, i.e. compressive instead of tensile.

As the diagonals of the square are at angles of 45 degrees with the four sides, then from the foregoing consideration we see that a pure shear stress is equivalent to two normal stresses of equal intensity acting in directions at 45 degrees to it, one of the normal stresses being tensile and the other compressive, the two latter acting in directions at right angles to each other.

The correctness of the above conclusions is again confirmed by practical experience, for it is found that if a shaft composed of some material which is stronger in shear than in tension (cast iron for example) be twisted until it fails, failure occurs in tension, the fracture being an oblique one in the form of a screw surface, the angle of the helix being approximately 45 degrees.

49.—**Complex Stress.** The simplest kind of stress is that which acts in one direction only, such stress being produced in rods exposed to simple tensile forces or in short columns exposed purely to compressive forces. In these cases, the stress acts in a longitudinal direction only. A complex state of stress is produced in a body when a number of stresses act in different directions simultaneously. A rectangular bar, for example, may have longitudinal forces applied to its ends, and, in addition, transverse forces applied either to one or two pairs of opposite sides.

When a body is in a complex state of stress, the actual stresses at any point may be shown to be precisely the same as those which would be produced by a combination of three simple stresses, either tensile or compressive, acting in directions mutually at right angles to one another. In other words, the complex state of stress produced at a point in a body by the application of a number of forces acting in different directions may be produced by three simple tensile or compressive stresses acting mutually at right angles to one another.

These simple stresses are termed *principal stresses*, and the directions in which they act are termed the *principal axes of stress*.

The tensile and compressive stresses which we found a simple shear stress to be equivalent to in connection with Fig. 11 are principal stresses, the two acting at right angles to each other. The third principal stress in this case is zero.

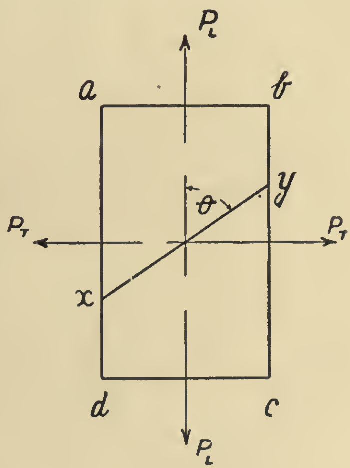

Fig. 12

Suppose *abcd* in Fig. 12 to represent a block of material subjected to a longitudinal pull P_L and a transverse pull P_T, and suppose we require to find the resultant stress acting on the plane *xy* inclined at some angle θ to the axis.

Let the normal stress on any horizontal plane due to the longitudinal pull P_L be represented by f_L, and the normal stress on any vertical plane due to the transverse pull P_T by f_T. Proceeding in the same way as we did when finding the normal and tangential stresses acting over the oblique section of Fig. 9, it is easy to show that the normal stress on the section *xy* of Fig. 12 due to the load P_L is $f_L \sin^2\theta$, whilst the tangential stress due to the same load is $f_L \sin\theta \cos\theta$.

Similarly, the normal stress over the section due to the load P_T is $f_T \cos^2\theta$ and the tangential stress $-f_T \sin\theta \cos\theta$.

Thus,

Let P_{nL} = the normal load due to P_L.
,, f_{nL} = ,, normal stress due to P_L.
,, P_{tL} = ,, tangential load due to P_L.
,, f_{tL} = ,, tangential stress due to P_L.
,, P_{nT} = ,, normal load due to P_T.
,, f_{nT} = ,, normal stress due to P_T.
,, P_{tT} = ,, tangential load due to P_T.
,, f_{tT} = ,, tangential stress due to P_T.
,, A_h = area of horizontal section.
,, A_o = ,, of oblique section.
,, A_v = ,, of corresponding vertical section

Then $\dfrac{A_h}{A_o} = \sin\theta$, and $A_o = \dfrac{A_h}{\sin\theta}$

$$f_{nL} = \frac{P_{nL}}{A_o} = \frac{P_L \sin\theta}{\dfrac{A_h}{\sin\theta}} = \frac{P_L \sin^2\theta}{A_h} = f_L \sin^2\theta$$

$$f_{tL} = \frac{P_{tL}}{A_o} = \frac{P_L \cos\theta}{\dfrac{A_h}{\sin\theta}} = \frac{P_L \cos\theta \sin\theta}{A_h} = f_L \sin\theta \cos\theta.$$

Again, $\dfrac{A_v}{A_o} = \cos\theta$, and $A_o = \dfrac{A_v}{\cos\theta}$

$$f_{nT} = \frac{P_{nT}}{A_o} = \frac{P_T \cos\theta}{\dfrac{A_v}{\cos\theta}} = \frac{P_T \cos^2\theta}{A_v} = f_T \cos^2\theta$$

$$f_{tT} = \frac{P_{tT}}{A_o} = \frac{P_T \sin\theta}{\dfrac{A_v}{\cos\theta}} = \frac{P_T \sin\theta \cos\theta}{A_v} = f_T \sin\theta \cos\theta.$$

The tangential stress due to P_T acts in the opposite direction to the tangential stress due to P_L, and must consequently be written $-f_T \sin\theta \cos\theta$.

The total normal stress due to both loads is therefore $f_L \sin^2\theta + f_T \cos^2\theta$, and the total tangential stress $f_L \sin\theta \cos\theta + (-f_T \sin\theta$

$\cos\theta)=(f_L-f_T)\sin\theta\cos\theta$. To find the resultant stress over the section, and the angle at which this stress acts to the section, it is necessary to combine the total normal and tangential stresses. This may be done either mathematically or by a geometrical construction. We will consider first how the two may be combined mathematically.

As the normal and tangential stresses act in directions at right angles to each other, the triangle of forces in this case would be a right-angled triangle, and the resultant stress would be represented by the hypotenuse of the triangle. Remembering that the sum of the squares of the shorter sides of a right-angled triangle are together equal to the square of the hypotenuse, the latter is equal to the square root of the sum of the squares of the shorter sides. One of the shorter sides will represent the total normal stress, which we will again denote by f_n, and the other side will represent the total tangential stress, which we will represent by f_t. The resultant stress, f, will then be represented by the hypotenuse.

We have now this relation: $f=\sqrt{f_n^2+f_t^2}$.

Substituting the above values of f_n and f_t

$$f=\sqrt{(f_L\sin^2\theta+f_T\cos^2\theta)^2+\{(f_L-f_T)\sin\theta\cos\theta\}^2}$$

$$=\sqrt{f_L^2\sin^4\theta+2f_L\sin^2\theta f_T\cos^2\theta+f_T^2\cos^4\theta+f_L^2\sin^2\theta\cos^2\theta-2f_L\sin^2\theta f_T\cos^2\theta+f_T^2\sin^2\theta\cos^2\theta}$$

$$=\sqrt{f_L^2(\sin^4\theta+\sin^2\theta\cos^2\theta)+f_T^2(\cos^4\theta+\sin^2\theta\cos^2\theta)}$$

$$=\sqrt{f_L^2\sin^2\theta(\sin^2\theta+\cos^2\theta)+f_T^2\cos^2\theta(\cos^2\theta+\sin^2\theta)}.$$

Now $\sin^2\theta+\cos^2\theta=1$.

Therefore, $$f=\sqrt{f_L^2\sin^2\theta+f_T^2\cos^2\theta}.$$

If we let ϕ represent the angle which the direction of the resultant stress f makes with the section xy, then the angle may be obtained from the relation

$$\tan\phi=\frac{f_n}{f_t}=\frac{f_L\sin^2\theta+f_T\cos^2\theta}{(f_L-f_T)\sin\theta\cos\theta}.$$

Dividing numerator and denominator by $\cos^2\theta$, we have

$$\tan\phi=\frac{f_L\dfrac{\sin^2\theta}{\cos^2\theta}+f_T\dfrac{\cos^2\theta}{\cos^2\theta}}{(f_L-f_T)\dfrac{\sin\theta\cos\theta}{\cos^2\theta}}=\frac{f_L\tan^2\theta+f_T}{(f_L-f_T)\tan\theta}.$$

$$\therefore\ \underline{f=\sqrt{f_L^2\sin^2\theta+f_T^2\cos^2\theta}}.$$

$$\underline{\tan\phi=\frac{f_L\tan^2\theta+f_T}{(f_L-f_T)\tan\theta}}$$

50.—Ellipse of Stress. The above method of combining the total normal and tangential stresses acting on the section *xy* of Fig. 12 is somewhat cumbersome, so the geometrical construction may be described.

Take any point *O* (see Fig. 13) as centre, and describe two circles, one of radius *Oy* equal to f_L, and the other of radius *Ox* equal to f_T, any convenient scale being chosen. From the centre *O* draw a radius *Or* to make an angle θ with the vertical centre

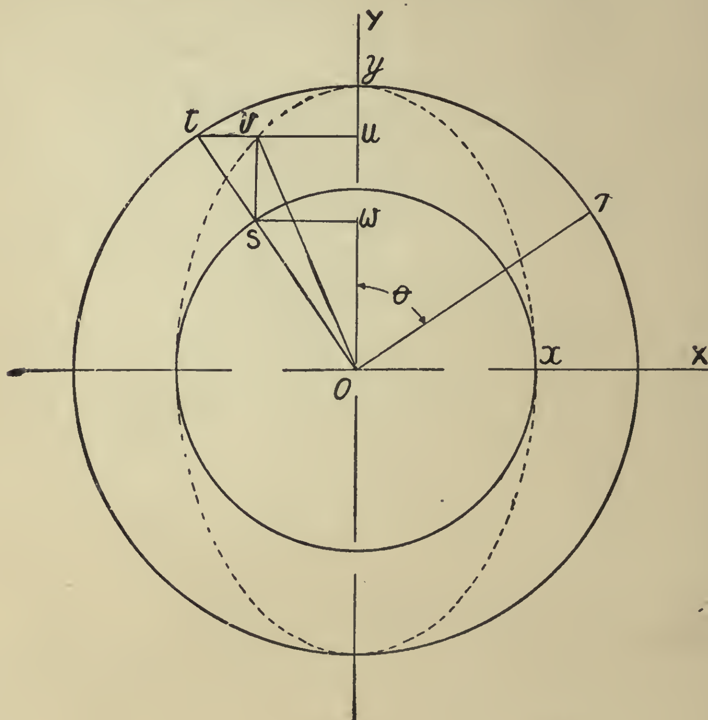

Fig. 13

line *OY*. Now draw a line from the centre *O* at right angles to *Or* to cut the small circle in point *s* and the large one in point *t*. Draw a perpendicular from *t* to meet the vertical centre line in the point *u*, and from point *s* perpendiculars to meet *tu* in the point *v* and the vertical centre line in point *w*. Join *Ov*.

Now in the right-angled triangle *Ovu*,

$$Ov = \sqrt{Ou^2 + uv^2}.$$

But $\dfrac{Ou}{Ot} = \cos(90 - \theta)$, $\therefore$ $Ou = Ot \cos(90 - \theta) = f_L \sin\theta$.

Also $\dfrac{ws}{Os} = \sin(90 - \theta)$, from which $ws = Os \sin(90 - \theta) = f_T \cos\theta$.

Now $ws=uv$, $\therefore$ $uv=f_T \cos\theta$. Substituting these values of Ou and uv in the equation,

$$Ov=\sqrt{Ou^2+uv^2}, \quad \text{we obtain}$$

$$Ov=\sqrt{f_L{}^2 \sin^2\theta+f_T{}^2 \cos^2\theta}.$$

We have shown (page 51) that the resultant stress,

$$f=\sqrt{f_L{}^2 \sin^2\theta+f_T{}^2 \cos^2\theta}.$$

It follows therefore that Ov in Fig. 13 represents the stress f in magnitude.

That Ov also represents the stress in direction as well as in magnitude may be proved as follows:—

Let the angle vOu be represented by α.

Then $\qquad \phi=\alpha+\theta.$

Also, $\qquad \tan\alpha=\dfrac{vu}{Ou}=\dfrac{sw}{Ou}=\dfrac{sw}{Ow}\cdot\dfrac{Ow}{Ou}$

$$=\frac{sw}{Ow}\cdot\frac{Os}{Ot}=\tan(90-\theta)\frac{Os}{Ot}=\cot\theta\frac{f_T}{f_L}.$$

Now if $\phi=\alpha+\theta$, we know that

$$\tan\phi=\frac{\tan\alpha+\tan\theta}{1-\tan\alpha\tan\theta}.$$

Substituting in this expression the value of $\tan\alpha$ above found, we get

$$\tan\phi=\frac{\dfrac{f_T}{f_L}\cot\theta+\tan\theta}{1-\cot\theta\dfrac{f_T}{f_L}\tan\theta}=\frac{\dfrac{f_T}{f_L}\cot\theta+\tan\theta.}{1-\dfrac{f_T}{f_L}}.$$

Multiplying numerator and denominator by $f_L \tan\theta$,

$$\tan\phi=\frac{f_T+f_L\tan^2\theta}{f_L\tan\theta-\dfrac{f_T}{f_L}\cdot f_L\tan\theta}=\frac{f_T+f_L\tan^2\theta}{(f_L-f_T)\tan\theta}.$$

$$\therefore\ \tan\phi=\frac{f_L\tan^2\theta+f_T}{(f_L-f_T)\tan\theta}, \text{ as found previously.}$$

The above construction may be repeated to find the resultant stress, due to a longitudinal pull P_L and a transverse pull P_T, over a section inclined at any other angle to the axis. If the construction be applied for a number of values of θ so as to find a number of points corresponding to v, and a curve be drawn through the various points, it will be found that this curve is an ellipse, the semi-axes of which are Oy and Ox, equal respectively to f_L and f_T. This ellipse is known as the *ellipse of stress*, and when once it has been constructed the resultant stress on any section at any angle θ to the axis may be readily obtained by drawing a radius from the centre O, at right angles to the direction of the section, to cut the outer circle, then drawing from the point of intersection a line perpendicular to OY to cut the ellipse in a certain point, and from the latter point drawing a line to meet the centre of the circle; this line then represents, to the scale chosen in setting out the construction, the resultant stress required, and the angle it makes with the direction of the section is the angle at which the resultant stress acts to the section. Thus, Ov in Fig. 13 represents in magnitude and direction the resultant stress on the oblique section xy in Fig. 12 inclined at an angle θ to the axis, due to the longitudinal pull P_L, and the transverse pull P_T acting simultaneously, and the angle rOv shows the angle which the line of action of the stress makes with the section.

The semi-axes of the ellipse of stress represent the principal stresses, and their directions are the principal axes of stress. Note that if the stresses f_L and f_T be equal to each other, the ellipse becomes a circle.

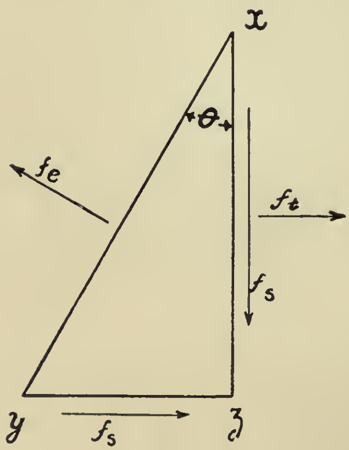

Fig. 14

51.—**Shear Stress in combination with Tensile or Compressive Stress.** In actual practice, cases are sometimes met with where a shear stress acts in combination with a tensile or a compressive stress. A shaft, for instance, may be exposed to shear in consequence of the turning moment acting upon it, whilst it may also be exposed to tension or compression at the same time due to the bending action caused by the weight of pulleys, fly-wheels, etc. For this reason, it is necessary to know how to determine the equivalent tensile or compressive stress in the material.

Let the triangle xyz of Fig. 14 represent a small element subjected to a shearing stress f_s and a tensile stress f_t along the side xz.

We require to find an equivalent stress, f_e, which, acting along the face xy, would balance the stresses f_s and f_t acting along the face xz.

It has been already shown that a shear stress is always accompanied by another shear stress of equal intensity acting at right angles to it, so it follows that there will also be a stress f_s acting along yz.

The total stress acting along yz, viz. $f_s yz$, together with the total stress due to f_t acting over the face xz, viz. $f_t xz$ will be balanced by the horizontal component of the total stress acting over the face xy. The latter is $f_e \cos \theta xy$, where θ is the angle between the sides xy and xz.

Expressing this as an equation, we have,

$$f_s yz + f_t xz = f_e \cos \theta xy. \qquad (1)$$

Similarly, the total stress along xz due to f_s, viz. $f_s xz$, will be balanced by the vertical component of the total stress acting on xy. The latter is $f_e \sin \theta xy$.

Hence $$f_s xz = f_e \sin \theta xy. \qquad (2)$$

Now divide equation (1) by xy.

Then $$f_s \frac{yz}{xy} + f_t \frac{xz}{xy} = f_e \cos \theta,$$

i.e. $$f_s \sin \theta + f_t \cos \theta = f_e \cos \theta.$$

Transposing, $$f_e \cos \theta - f_t \cos \theta = f_s \sin \theta$$

$$(f_e - f_t) \cos \theta = f_s \sin \theta. \qquad (3)$$

Divide equation (2) by xy.

Then $$f_s \frac{xz}{xy} = f_e \sin \theta,$$

i.e. $$f_s \cos \theta = f_e \sin \theta. \qquad (4)$$

Now divide equation (3) by equation (4) to eliminate the trigonometrical ratios.

$$\frac{(f_e - f_t) \cos \theta}{f_s \cos \theta} = \frac{f_s \sin \theta}{f_e \sin \theta}$$

$$\frac{f_e - f_t}{f_s} = \frac{f_s}{f_e}.$$

Cross-multiplying, $$f_e(f_e - f_t) = f_s^2$$

$$f_e^2 - f_e f_t = f_s^2.$$

This is an ordinary quadratic equation, which may be solved in the usual manner by completing the square.

$$f_e^2 - f_e f_t + (\tfrac{1}{2} f_t)^2 = f_s^2 + (\tfrac{1}{2} f_t)^2$$

$$= f_s^2 + \frac{f_t^2}{4}.$$

Extracting the square root of both sides of the equation,

$$f_e - \frac{f_t}{2} = \pm \sqrt{f_s^2 + \frac{f_t^2}{4}}$$

$$\therefore\ f_e = \frac{f_t}{2} \pm \sqrt{f_s^2 + \frac{f_t^2}{4}}.$$

Thus, according to the solution, f_e has two values, but the one we are concerned with is the greater one, so we take the value having the positive sign.

The final expression is therefore

$$\underline{f_e = \frac{f_t}{2} + \sqrt{f_s^2 + \frac{f_t^2}{4}}.}$$

52.—**Different forms of Strain.** In Chapter I. we considered the meaning of strain in connection with the stretching or crushing of bars and columns, where the strain was in the nature of an extension or shortening.

A strain may, however, be very different from a simple extension or shortening, and the measurement of other forms of strain has next to be considered.

Lateral Strain: Poisson's Ratio. When a rod or bar is extended, or a short column compressed, an alteration takes place in the lateral dimensions, and it is found that for any given material the transverse strain bears a certain ratio, generally denoted by $\frac{1}{\sigma}$, to the longitudinal strain, providing the limits of elasticity be not exceeded.

Thus,

$$\frac{\text{Transverse strain}}{\text{Longitudinal strain}}=\frac{1}{\sigma}.$$

This ratio is usually known as *Poisson's Ratio*, and its value for metals is generally between $\frac{1}{4}$ and $\frac{1}{3}$. When the force applied is so great that the limits of elasticity are exceeded, the ratio is different from that relating to elastic strain.

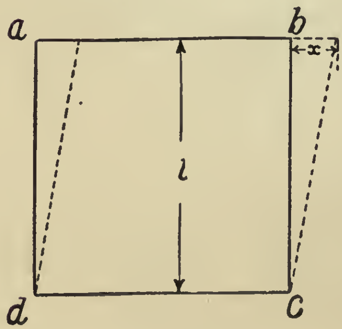

Fig. 15

Shearing Strain. In specifying a tensile strain in a rod, we measure the extension and divide this by the original length of the rod.

A shear strain may be measured in two ways.

Suppose *abcd* in Fig. 15 to represent an element subjected to shear, the base *dc* being supposed held rigid. The effect of the shearing action will be to distort the element into a parallelogram, as shown by the dotted lines. Then the distortion may be measured by the amount of slide, x, and the shearing strain by the fraction $\frac{x}{l}$, where l represents the dimension indicated in the figure.

When the element is distorted, the angles at the corners are altered, two opposite angles being enlarged and the other two reduced. This change of angle may be used to measure the strain. Expressing the angles in radians, each angle measures originally $\frac{\pi}{2}$ radians. After the distortion, two of the angles are increased by an amount which may be represented by ϕ, to $\frac{\pi}{2}+\phi$, the other two being reduced to $\frac{\pi}{2}-\phi$. The angle ϕ thus serves as a measure of the strain, and is termed the *angle of shear.*

Volumetric Strain. If a body be subjected either to pressure or tension over all its exterior surface, it will undergo a change of volume, without necessarily suffering distortion.

The strain produced in this way is generally termed *volumetric strain,* and if V represent the original volume of the body, and v the alteration of volume, the strain is measured by the ratio $\frac{v}{V}$.

53.—**Modulus of Transverse Elasticity or Rigidity.** It was shown in the first chapter that when a rod is subjected to a tensile load, the strain produced is directly proportional to the stress, within the limits of elasticity, and the stress bears a constant ratio to the strain, this ratio being termed the Modulus of Elasticity.

Similarly, it is found that when a body is subjected to shear, the shear strain is directly proportional to the stress. The ratio of stress to strain in this case is termed the *Modulus of Transverse Elasticity* or the *Modulus of Rigidity.* The letter G is commonly used to denote this constant.

Referring to Fig. 15, it is seen that shear strain may be measured by the fraction $\frac{x}{l}$, and *the modulus of rigidity is therefore the shear stress required to produce an amount of slide x equal to l.*

If the shear strain be measured by the angle of shear, ϕ, then if f_s represent the stress, the modulus of rigidity, being equal to stress divided by strain, will be denoted by $\frac{f_s}{\phi}$.

The modulus of rigidity is found to have a value equal to about two-fifths the value of the modulus of elasticity.

Modulus of Cubic Elasticity. The general law which states that strain is proportional to stress also holds good when the strain is one of volume. The ratio of stress to strain in this case is termed the *Modulus of Cubic Elasticity,* and it is generally denoted by the letter K.

54.—**Stress in bars of varying Section.** In all our calculations dealing with the tensile stress in a loaded bar, the stress has been found by dividing the total load by the cross sectional area of the bar, assuming the section to be uniform throughout and the stress to be distributed uniformly over the bar.

It is necessary now to inquire how the stress is affected if the bar be not of uniform section.

Consider a bar of the form shown in Fig. 16, where the section changes abruptly at X. If this bar were loaded with a tensile load sufficient to break it, fracture would most likely occur at X, unless the material were very ductile. If now we calculate the stress at a section just above X, and also at another section just below, by dividing the load by the respective sectional areas, the stress at the larger section would appear to be considerably less than that at the smaller one.

Fig. 16

If we assume the two sections to be indefinitely close together, a little consideration will serve to show that the real stress at X will be intermediate between the stress found by dividing the load by the area of the larger section and that found by dividing the load by the area of the smaller section. The distribution of stress where the abrupt change of section occurs is very unequal, and it is evident therefore that the usual method of calculating the stress cannot be correctly applied for sections very close to parts where there is an abrupt change of section, the stress not being uniformly distributed.

It so happens, however, that in materials generally the stress rapidly distributes itself, with the result that the distribution is practically uniform at a short distance from the change of section.

In order to distribute the stress uniformly over a bar or rod which must change in section, the obvious thing to do is to arrange for the section to change very gradually, and this is done by rounding off the corners with as large a radius as practicable, as indicated by dotted curves in Fig. 16.

When the change of section is gradual, the stress at any section may safely be calculated by dividing the area of the section into the load, and the result will be sufficiently accurate for all practical purposes.

Referring to the bar of Fig. 16, fracture due to a heavy load would, as already stated, most likely occur at X, *unless the material were very ductile.* If the material were sufficiently ductile, fracture would most probably occur well away from X; the reason for this being that with plastic material the tendency is for the fracture to take place at a point where the section is most free to contract. At the section X the overlapping or projecting metal tends to prevent the reduction of area which precedes failure of the bar.

55.—**Stress in Compound Bars.** Cases are sometimes met with in practice where a bar of a certain material is combined with a bar of some other material, and it is then required to find the stress in each bar, due to a certain load on the combined bar.

Let the two bars be denoted by 1 and 2 respectively.

Let A_1 represent the cross sectional area of 1.

,, A_2 ,, ,, ,, ,, 2.

,, E_1 represent the modulus of elasticity of the material of which bar 1 is made.

Let E_2 represent the modulus of elasticity of the material of which bar 2 is made.

Let F be the load on the combined bar.

,, P be the load taken by bar 1.

Then $F-P$=load taken by bar 2.

Let f_1 be the stress in bar 1 due to the load P.

,, f_2 ,, ,, 2 ,, ,, $(F-P)$.

Then
$$f_1=\frac{P}{A_1},$$
$$f_2=\frac{F-P}{A_2}.$$

Now,
$$\text{Strain}=\frac{\text{stress}}{\text{modulus of elasticity}}.$$

$$\text{Strain in bar } 1=\frac{f_1}{E_1}$$
$$\text{,, ,, } 2=\frac{f_2}{E_2}.$$

As the two bars are combined together, the amount of stretch, and therefore the strain, must be the same for each bar. Hence
$$\frac{f_1}{E_1}=\frac{f_2}{E_2}.$$

Substituting the values of the stresses,
$$\frac{\frac{P}{A_1}}{E_1}=\frac{\frac{F-P}{A_2}}{E_2}, \text{ or } \frac{P}{A_1E_1}=\frac{F-P}{A_2E_2}.$$

From this, by cross-multiplying,
$$PA_2E_2=(F-P)A_1E_1$$
$$=FA_1E_1-PA_1E_1$$
$$PA_2E_2+PA_1E_1=FA_1E_1$$
$$\therefore P=\frac{FA_1E_1}{A_2E_2+A_1E_1}$$

or
$$P=\frac{F}{1+\frac{A_2E_2}{A_1E_1}}.$$

In a similar manner, it can be shown that the load taken by bar 2, which load may be called Q, is,

$$Q=\frac{FA_2E_2}{A_1E_1+A_2E_2},$$

or

$$Q=\frac{F}{1+\frac{A_1E_1}{A_2E_2}}.$$

Having found the load taken by each bar, the stress in either bar is readily found by dividing the load by the cross sectional area of the bar.

Thus, $$f_1=\frac{P}{A_1}=\frac{FA_1E_1}{A_2E_2+A_1E_1}\div A_1=\frac{FE_1}{A_2E_2+A_1E_1},$$

and $$f_2=\frac{F-P}{A_2}=\frac{Q}{A_2}=\frac{FA_2E_2}{A_1E_1+A_2E_2}\div A_2=\frac{FE_2}{A_1E_1+A_2E_2}.$$

It will be understood that these results are only correct so long as the limits of elasticity are not exceeded.

56.—**Work done in Stretching a Bar: Resilience.** Mechanical work is said to be performed whenever a resistance is overcome through space, the amount of the work being measured by the product of the mean resistance and the space passed through. When a bar is subjected to tensile loads in a testing machine, work is done on the bar in stretching it. In the ordinary load-strain diagram, the ordinates represent the loads or the resistance offered by the bar to being stretched, and the abscissæ the extensions or the space through which the resistance is overcome. It is clear then that the area of the load-strain diagram will represent the work done on the bar.

The capacity of a body for resisting live loads and shocks may be judged by the amount of work done upon it in stretching it up to the point of fracture; the greater the work done, the better able is the material to resist the loads and shocks.

It is a well-known fact that bolts used for fastening parts together which are exposed to vibration and shock, as, for example, gas engine and steam engine connecting rod bolts, are usually turned down for the greater part of their length to a diameter equal to that at the bottom of the screw thread, as by so doing the tendency to fracture in the thread is greatly reduced. The reason for this is that if the bolts be not turned down, the straining is mainly confined to the reduced sections at the bottom of the screw threads, so that the work done in stretching the bolts is comparatively small. If, however, the bolts be turned down to a diameter equal to that at the bottom of the thread, the straining is distributed over a comparatively great length, and the work done in stretching will therefore be considerably greater than it would otherwise be.

According to Hooke's Law, the amount of stretch of a bar subjected to tensile force is directly proportional to the stress producing it, within the limits of elasticity. That portion of the load strain curve up to the elastic limit will therefore be a straight line, and the work done in stretching the bar up to the elastic limit will consequently be represented by a triangle.

Suppose the bar is extended by an amount x when the elastic limit is reached, the load or tensile force on the bar then being F, and the load having been gradually applied.

If we let W represent the work done in stretching the bar up to the elastic limit, then

$$W = \text{mean resistance} \times \text{space overcome.}$$
$$= \frac{F}{2} \times x.$$

If f be the stress in the bar when the elastic limit is reached, and A the sectional area of the bar, then $f = \frac{F}{A}$, from which $F = fA$.

Again, $\frac{\text{stress}}{\text{strain}} = \text{modulus of elasticity, E,}$

i.e. $\frac{f}{\frac{x}{l}} = E$, where l=original length of the bar, $\frac{x}{l}$ being the strain.

Hence, $$x = \frac{fl}{E}.$$

Substituting for F and x in the equation, $W = \frac{F}{2} \times x$, we get

$$W = \frac{fA}{2} \times \frac{fl}{E} = \frac{f^2}{E} \times \frac{Al}{2}.$$

It will be noticed that Al represents the volume of the bar, so that

$$W = \frac{f^2}{E} \times \tfrac{1}{2} \text{ volume of bar.}$$

The work done in stretching a bar is therefore proportional to the volume or the weight of the bar.

The term *resilience* is employed to denote the work done in stretching a bar *up to the elastic limit.*

The above expression for the work done holds just the same if the bar be stretched to some point below the elastic limit, but f will then represent not the stress at the elastic limit, but some lower stress, i.e. that corresponding to the strain.

57.—**Effect of a Live Load in Stretching a Bar.** The work done by a load F gradually applied to extend a bar by an amount x is equal to the *mean* load multiplied by the extension. As the mean load is $\frac{F}{2}$, it follows that a load of this amount applied suddenly, but without initial velocity, would stretch the bar the same amount as a load F applied gradually. If now the load F were applied suddenly, the extension would be doubled, and the work done (the mean resistance multiplied by the strain) would be $F \times 2x$. Hence,

$$\frac{\text{Work done by live load}}{\text{Work done by dead load}} = \frac{F \times 2x}{\frac{F}{2} \times x} = \frac{4}{1}.$$

Thus, when the load is applied suddenly (without initial velocity, it must be noted), the amount of work done in stretching a bar is four times that done when the load is applied very gradually.

58.—**Stresses caused by Live Loads.** In actual practice, it frequently happens that the load is applied suddenly, and it is important to consider how the resulting stress compares with that which would result if the load were applied gradually.

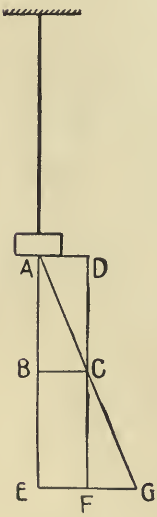

Fig. 17

The general effect is best seen by taking the case of an elastic string loaded first by a weight applied very gradually, and then by the same weight applied suddenly.

With regard to the gradually applied load, which may be called F, the actual load on the string to commence with is nil, but it is increased by very small increments until the full value of F is reached. If the extensions and the actual values of the weights producing them be plotted together, the resulting curve will of course be a straight line. Thus, AC in Fig. 17 is the curve showing the relation between the load and the extension, BC representing the load F, and AB the extension.

Now let the load F be applied suddenly. When, in consequence, the string has been extended an amount AB, the work given out by the load in falling from A to B is represented by the rectangle $ABCD$. The work done in stretching the string is, however, represented by the triangle ABC, and this is seen to be only one-half the work given out by the falling load, the area of the triangle being one-half that of the rectangle. It follows then, that an amount of energy represented by the triangle ACD is available for stretching the string still further. Produce the vertical DC to the point F and the load extension curve AC to the point G, the points F and G being fixed so that the triangle CFG is equal to the triangle ACD. The final extension is then represented by AE, and this is clearly equal to twice the extension AB. Within the limits of elasticity, we know that if we wish to double the strain or the extension, we must double the stress or the load. Hence, as the load EG is twice BC, the stress in the string when stretched the amount AE is twice what it is when the string is stretched the amount AB. The extension AB is produced by the load F gradually applied, and the extension AE by the same load suddenly applied. *Therefore the stress due to a live load is just twice the stress due to a dead load of the same amount.* This shows the importance of applying the load on a machine or structure as gradually as possible.

59.—**Stresses caused by Impulsive Loads.** An impulsive load must not be confused with a "live" load. The latter is a load suddenly applied, but without any initial velocity, whilst an impulsive load is one applied with an initial velocity. Take the case of a load on a crane hook. If the load be lifted on to the hook and suddenly released it is a "live" load, but if it be dropped on to the hook it will possess an initial velocity at the moment it reaches the hook, and it will then be an impulsive load. It is not difficult to understand that an impulsive load will give rise to greater stresses than a "live" load of the same magnitude.

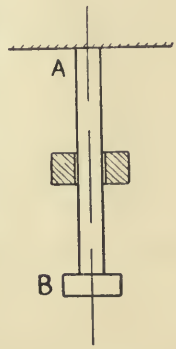

Fig. 18

Suppose AB in Fig. 18 to represent a bar of metal secured at its upper end to some convenient point overhead and provided at its lower end with a collar, and suppose a sliding weight to be placed between the upper end and the collar.

If now the weight be allowed to fall freely through a certain height on to the collar, it will give up a certain amount of energy, which will be used up in extending or doing work upon the bar.

Let F be the weight.

" h " " height through which F falls before reaching the collar.

Let x " " extension produced in the bar.

Then the work given up by the falling weight will be F $(h+x)$. It is known from our previous work that when a bar is extended within the limits of elasticity, the work done on it is equal to $\frac{f^2}{E} \times \frac{1}{2}$ volume of bar, f representing the stress corresponding to the strain produced.

We have then the equation,

$$F(h+x)=\frac{f^2}{E}\times\frac{V}{2}, \text{ where } V=\text{volume of bar.}$$

In most cases the extension x is small, in fact negligible, in comparison with the height h, and if it be neglected the equation becomes $Fh=\frac{f^2}{E}\cdot\frac{V}{2}$, from which the stress $f=\sqrt{\frac{2FhE}{V}}$.

The volume $V=Al$, where A=cross sectional area and l=the original length of the bar.

Substituting, $f=\sqrt{\frac{2FhE}{Al}}$.

If x be not neglected, the stress f may be found in the following manner:—

The general equation is,

$$F(h+x)=\frac{f^2}{E}\times\frac{V}{2}.$$

Now $x=\frac{lf}{E}$, so substituting in the general equation, we have

$$F\left(h+\frac{lf}{E}\right)=\frac{f^2}{E}\times\frac{V}{2},$$

$$Fh+\frac{Flf}{E}=\frac{f^2}{E}\cdot\frac{V}{2}.$$

Rearranging, $$\frac{f^2}{E}\cdot\frac{V}{2}-\frac{Flf}{E}=Fh.$$

Divide throughout by $\frac{V}{2E}$.

Then $$f^2-\frac{2Flf}{V}=\frac{2FhE}{V}.$$

Completing the square,

$$f^2-\frac{2Flf}{V}+\left(\frac{Fl}{V}\right)^2=\frac{2FhE}{V}+\frac{F^2l^2}{V^2}.$$

Extracting the square root,

$$f-\frac{Fl}{V}=\pm\sqrt{\frac{2FhE}{V}+\frac{F^2l^2}{V^2}}$$

$$=\pm\sqrt{\frac{2FhEV+F^2l^2}{V^2}}.$$

$$\therefore f=\frac{Fl}{V}\pm\frac{\sqrt{2FhEV+F^2l^2}}{V}.$$

Substituting Al for V, the final expression becomes

$$f=\frac{F}{A}+\frac{\sqrt{2FhEAl+F^2l^2}}{Al}.$$

We take, of course, the positive sign in the expression, as this gives us the maximum stress. It is important to note that the above results are only correct if the strain which the bar suffers be within the limits of elasticity of the material. If the strain be beyond this, the work done in straining the bar may be obtained by measuring up the area of the stress-strain curve, and this area will be equal to $F(h+x)$.

60.—**Repeated Stresses: Wöhler's Experiments.** A brief reference was made to the effect of repeated stresses in Chapter I. in connection with the explanation of the term "fatigue." It was there pointed out that a bar of iron or steel may be broken with a load less in amount than the true breaking load by applying and removing the load repeatedly. If the load applied be nearly equal to the breaking load, then a few applications and removals will suffice to cause failure; but if it be but a small fraction of the breaking load, then an indefinite number of applications and removals may not produce any appreciable weakening.

A very exhaustive series of experiments, extending over a period of twelve years, was carried out many years ago by the engineer, Wöhler, on behalf of the Prussian Ministry of Commerce, to show the effect of repeated stresses on iron and steel. In these experiments, Wöhler employed some ingeniously contrived machines which enabled him to submit test pieces to tensile, compressive, and torsional stresses of varying amounts, some of the variations being from zero to a maximum in tension or compression, others from a maximum of one kind of stress to a maximum of the opposite kind, and so on.

The general nature of the results of the experiments may be gathered from the following tables:—

WÖHLER'S EXPERIMENTS ON BARS SUBJECTED TO REPETITIONS OF TENSILE STRESSES FROM A MAXIMUM TO A MINIMUM.

Material, Cast Iron from Locomotive Cylinder.

Stress in tons per square inch. Maximum.	Minimum.	No. of repetitions before fracture
7·62	0	3,140
6·69	0	4,000
6·22	0	10,342
5·73	0	45,028
5·26	0	78,682
5·03	0	27,885
5·03	0	35,599
4·78	0	208,439
4·78	0	7,200,000, unbroken.
4·78	0	7,600,000 ,,

Material, Axle Steel.

Stress in tons per square inch. Maximum.	Minimum.	No. of repetitions before fracture.
38·20	0	18,741
33·40	0	46,286
28·65	0	170,170
26·14	0	123,770
23·87	0	473,766
22·92	0	13,600,000, unbroken.
21·95	0	13,200,000 ,,

BAR SUBJECTED TO REPETITIONS OF TRANSVERSE STRESSES FROM A MAXIMUM OF ONE KIND TO A MAXIMUM OF THE OPPOSITE KIND.

Material, Iron for Axles.

Stress in tons per square inch. Maximum.	Minimum.	No. of repetitions before fracture.
15·3	−15·3	56,430
14·3	−14·3	99,000
13·4	−13·4	183,145
12·4	−12·4	479,490
11·5	−11·5	909,840
10·5	−10·5	3,632,588
9·6	− 9·6	4,917,992
8·6	− 8·6	19,186,791
7·6	− 7·6	132,250,000, unbroken.

The tabulated results of the experiments clearly show how the number of applications and removals of the load required to break the test piece depends on the range of stress; by keeping the range within a certain limit, the number of repetitions required to produce failure may be rendered indefinitely great.

Wöhler himself summarised the results of all the tests in a general law, usually known as *Wöhler's Law*: "Rupture of material may be caused by repeated vibrations, none of which attains the absolute breaking limit. The differences of the limiting strains are sufficient for the rupture of the material."

Wöhler further gave the values of the stresses which he considered might be imposed an indefinite number of times upon wrought iron and steel members without appreciably weakening the materials. Thus, for wrought iron in tension, the stress might vary from +8·35 to +·013 tons per square inch, and for cast steel in tension from +15·30 to +5·10 tons. For wrought iron subjected alternately to tension and compression, the stress might vary from +2·82 to −2·82 tons per square inch, and for cast steel, from +5·58 to −5·58 tons per square inch. The plus sign, it may be mentioned, signifies tension, and the minus sign compression.

The figures in the following table were also given to show the stresses and the range of stress which would only produce fracture of the materials experimented upon after an indefinitely large number of applications of the load. The figures apply to bars subjected to simple tension, compression, or bending.

	Maximum Stress.	Minimum Stress.	Range of Stress.
Wrought Iron . .	7·65	−7·65	15·3
,, ,, . .	15·8	0	15·8
,, ,, . .	21·0	11·50	9·50
Cast Steel Axles . .	13·38	−13·38	26·76
,, ,, . .	23·0	0	23·0

As the stress which produces fracture in a bar subjected to repeated stresses depends on the range of the fluctuation of stress as well as upon the ultimate static strength of the material, several empirical formulæ have been devised to show the relation between the maximum stress which can be applied an indefinite number of times, the range of stress, and the ultimate static strength of the material. Of these, Gerber's equation, as given by Professor Unwin, is one of the best known.

This equation is,

$$f_{max.} = \frac{\Delta}{2} + \sqrt{f^1 - n\Delta f^1},$$

where $f_{max.}$ = the maximum stress which can be applied an indefinite number of times,

Δ = the range of stress,

and f^1 = the ultimate static strength of the material,

n = a constant, the value of which depends on the material.

For ductile iron or steel, n is, according to Professor Unwin, 1·5, whilst for the harder qualities it is 2·0.

Other authorities, notably Bauschinger and Sir B. Baker, have since carried out experiments similar to those of Wöhler, and their results confirm the conclusions arrived at by him.

The fact that materials are liable to become weaker if subjected to varying and repeated loads should obviously not be disregarded, as it frequently is, when deciding upon a factor of safety.

Several formulæ have been devised for determining the working stress which should be adopted for any member of a machine or structure exposed to repeated stresses. Amongst these, probably the formula of Launhardt and Weyrauch is the most popular. This is,

$$\text{Actual working stress} = \frac{f}{1 \cdot 5}\left(1 + \frac{\text{minimum load}}{2 \times \text{maximum load}}\right),$$

where f is the dead load working stress.

As an example of the application of this formula, suppose some member of a machine to be exposed to a dead load of 6 tons, and suppose in addition the member is exposed alternatively to a tensile load of 6 tons and a compressive load of 3 tons.

The minimum load is then $6-3=3$ tons, the maximum $6+6=12$ tons, whilst the dead load stress is of course f tons per square inch.

Hence,

$$\text{Actual working stress} = \frac{f}{1 \cdot 5}\left(1 + \frac{3}{2 \times 12}\right) = \frac{f}{1 \cdot 5} \times 1 \cdot 125 = \underline{\cdot 75 f.}$$

The actual working stress should therefore be three-quarters the safe stress which would be adopted if the load were a dead or steady load.

In the experiments carried out by Wöhler, the stresses were imposed at the rate of approximately 60 per minute throughout, no attempt being made to ascertain if the rate of repetition had any effect on the general results of the tests. Later experimenters have therefore made further tests to ascertain if the conclusions arrived at by Wöhler were materially affected by the rate of repetition. Messrs. Smith and Reynolds applied tensile and compressive stresses at rates varying from 1400 to 2500 per minute, and from their results they concluded that the number of repetitions of stress which a material such as iron or steel is capable of withstanding diminishes as the speed increases.

Messrs. Stanton and Bairstow, who also subjected specimens to reversals of direct stress, came to the conclusion that an alteration of the rate of repetition from 60 to 800 per minute had no marked effect on the results obtained. Another important conclusion they arrived at was that the limiting stress which can be borne by iron and steel depends on the range of stress, and, within fairly wide limits, is almost independent of the actual value of the maximum stress.

61.—**Stresses due to the Forces of Expansion and Contraction caused by Heating and Cooling.** It is a well-known fact that most metals expand when heated and contract when cooled. The forces of expansion and contraction are practically irresistible, and in the case of certain structures exposed to changes of temperature these forces must be duly provided against.

In a range of steam pipes, for example, it is found that the amount of expansion which occurs when steam is turned into the cold pipes is approximately $2\frac{1}{2}$ inches per 100 feet of length, the actual amount depending, of course, on the temperature of the steam, the material of which the pipes are made, etc. Unless suitable provision be made for taking up the expansive movements, the forces of expansion are such that the pipes may be seriously strained and even fractured, particularly if of cast iron, by the stresses set up in the material.

As another example, a horizontal steam engine may be cited. When steam is turned into the cylinders a certain amount of expansion occurs, and if the cylinders be bolted rigidly to the frame or bedplate, so that the movement cannot take place freely, fracture of the feet or other parts may possibly result.

Provision against this danger is therefore made when really necessary, by screwing up the nuts of the holding down bolts against a ferrule in such a manner that whilst the feet cannot be lifted from their seats by the movements of the engine, they are yet free to move in a longitudinal direction.

Very dangerous stresses may be, and sometimes are, set up in steam boilers by the forces of contraction which result from sudden cooling of the hot plates. Thus a fireman will frequently blow off the water and steam under pressure, and then, with the object of cooling down the boiler quickly for cleaning or inspection purposes, will play over the plates with cold water from a hose pipe. The sudden contraction which results from this cannot take place freely, the boiler being more or less a rigid structure, and, in consequence, the plates become very highly stressed, so much so in fact that fracture may occur.

Fortunately, in most cases, the engineer is not only able to make suitable provision against the forces of expansion and contraction, but he is frequently able to put them to practical advantage. Thus, the walls of a building which have become bulged have frequently been straightened by passing bolts, provided with large nuts and washers, through them, heating the bolts through a certain range of temperature, screwing up the nuts until the washers bear against the walls, allowing the bolts to contract, and repeating the operation one or more times as required. As contraction takes place an enormous force is exerted upon the walls, which are in consequence eventually straightened.

Similarly, contractile forces are employed in the manufacture of built-up guns (which are constructed by shrinking a number of concentric rings, one on another), and in securing cranks to crank shafts, etc.

It is a simple matter to calculate how much a bar of any particular metal expands or contracts on being heated or cooled through a given range of temperature. From experiment, we know with considerable accuracy what fraction of its length at 32 degrees Fahrenheit a bar of any metal will expand for every degree through which it is heated. This fraction is termed the *coefficient of expansion* of the metal. For steel, the coefficient is ·00000672, which means to say that a bar of steel of any uniform section expands ·00000672 of its length at 32 degrees F. for every degree through which it is heated; the bar would contract exactly the same amount for every degree it was cooled down. It is to be noted that the length of the bar referred to is the length at a certain temperature, viz. 32 degrees F. The fractional expansion of a bar at a temperature other than 32 degrees F. (per degree of temperature), would be very slightly different from the coefficient of expansion of the material, but the difference is so slight that we may base our calculations on the assumption that the fractional expansion per degree F. is the same, no matter what the temperature of the bar is to commence with. Knowing the range of temperature through which the bar is heated or cooled, then the expansion or contraction is obtained by multiplying the length by the coefficient and by the range of temperature.

Thus, let α=coefficient of expansion.
,, r=range of temperature.
,, x=amount of expansion or contraction.

Then $x=l\times\alpha\times r$.

The stress which may be set up in a metal bar when the bar is so fixed that the expansion or contraction cannot take place may be calculated in the following manner:—

Let f=stress in the material.
,, E=modulus of elasticity.

Then $E=\dfrac{f}{\frac{x}{l}}$, from which $f=\frac{x}{l}E$.

Now $x=l\times\alpha\times r$.

Substituting for x in the equation $f=\frac{x}{l}E$,

$$f=\frac{l\alpha rE}{l}=\underline{\alpha rE.}$$

It will be noticed that the length l cancels out, so that in finding the stress in any particular case the length need not enter into the question.

Worked Examples

62.—(1) **Normal and Tangential Stresses.** A stress of 10 tons per square inch acts at an angle of 30 degrees to a section in a strained solid. Find the stresses acting normally and tangentially to the section.

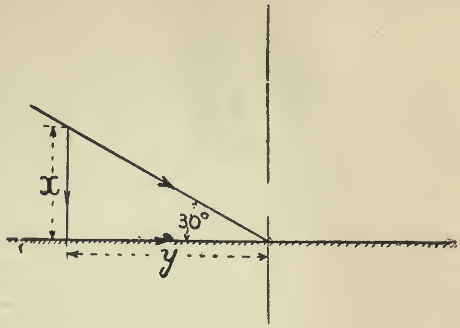

Fig. 19

Referring to Fig. 19, the stress is shown resolved into its normal and tangential components.

If we call x the normal component and y the tangential component, we have

$$\frac{x}{10}=\sin 30, \text{ and } \frac{y}{10}=\cos 30.$$

Hence, $x=10\sin 30=10\times\frac{1}{2}=\underline{5 \text{ tons per square inch.}}$

$y=10\cos 30=10\times{\cdot}866=\underline{8{\cdot}66 \text{ tons per square inch.}}$

Therefore, Normal stress $=\underline{5 \text{ tons per square inch.}}$

Tangential stress $=\underline{8{\cdot}66 \text{ tons per square inch.}}$

In the early part of the chapter we showed that the normal component was $f\cos\theta$, and the tangential component $f\sin\theta$, where θ was the angle between the normal to the section and the line of direction of the stress.

The angle θ in the present case would be $90-30=60°$.

Hence, Normal component $=f\cos 60=10\cos 60°$.

$=10\times{\cdot}5 =\underline{5 \text{ tons per square inch.}}$

Tangential component $=f\sin 60=10\times{\cdot}866$

$=\underline{8{\cdot}66 \text{ tons per square inch.}}$

(2) A vertical metal bar measuring 2 inches by 1 inch in cross section is pulled with a load of 12 tons. What are the normal and shearing stresses on a section which makes an angle of 40 degrees with the horizontal section?

Let f = the stress on sections at right angles to the axis of the bar.
,, f_n = the normal stress on the oblique section.
,, f_t = the tangential stress on the oblique section.
,, θ = the angle the oblique section makes with the horizontal section.

Then
$$f_n=f\cos^2\theta,$$
$$f_t=f\sin\theta\cos\theta.$$

Now $f=\dfrac{\text{load on bar}}{\text{area of section}}=\dfrac{12}{2}=6$ tons per square inch.

$f_n=f\cos^2\theta=6\times\cos^2 40=6\times\cdot766^2=$ 3·52 tons per square inch.

$f_t=f\sin\theta\cos\theta=6\times\sin 40\cos 40$

$=6\times\cdot643\times\cdot766=$ 2·95 tons per square inch.

∴ Normal stress on section = 3·52 tons per square inch.

Tangential or shear stress = 2·95 tons per square inch.

63.—**Shear Stress in combination with Tensile Stress.** (3) A rivet 1 inch diameter is exposed to a shearing force of 3·14 tons and a tensile force of 2·35 tons, the latter being caused by the contraction of the rivet on the plates. Find the maximum equivalent tensile stress in the rivet.

Let f_e = the equivalent tensile stress.

,, f_s = the shear stress.

,, f_t = the tensile stress.

Then,
$$f_e=\frac{f_t}{2}+\sqrt{f_s^2+\frac{f_t^2}{4}}$$

$$f_s=\frac{\text{shearing force}}{\text{area of section}}=\frac{3\cdot14}{\cdot785\times1^2}=\frac{3\cdot14}{\cdot785}=4 \text{ tons per square inch,}$$

$$f_t=\frac{\text{tensile force}}{\text{area of section}}=\frac{2\cdot35}{\cdot785\times1^2}=\frac{2\cdot35}{\cdot785}=\text{say, 3 tons per square inch,}$$

$$f_e=\tfrac{3}{2}+\sqrt{4^2+\frac{3^2}{4}}=1\cdot5+\sqrt{16+2\cdot25}$$

$$=1\cdot5+\sqrt{18\cdot25}=1\cdot5+4\cdot26=5\cdot76 \text{ tons per square inch.}$$

This represents the greater principal stress.

The minor principal stress would be

$$\frac{f_t}{2}-\sqrt{f_s^2+\frac{f_t^2}{4}}=1\cdot5-4\cdot26=-2\cdot76 \text{ tons per square inch.}$$

The minus sign indicates that the stress is compressive, whereas the greater principal stress is tensile.

64.—**Stress in a Compound Bar.** (4) A steel bar and a brass bar of the same length are securely attached together at each end, and the combined bar supports a load of 10 tons. If the steel bar has a section of 2 inches by 1, and the brass bar $2\times\frac{3}{4}$ inches, what is the load taken by each? Also, what is the stress in each bar? Moduli of elasticity, 30,000,000 lbs. per square inch for steel and 10,000,000 for brass.

Let P be the load taken by the steel bar.

Then 10−P is the load taken by the brass bar.

Stress on steel bar $=\dfrac{P}{2\times1}=\dfrac{P}{2}$ tons per square inch.

Stress on brass bar$=\frac{10-P}{2\times\frac{3}{4}}=\frac{10-P}{1\cdot5}$ tons per square inch.

$$\text{Strain}=\frac{\text{stress}}{\text{modulus of elasticity}}.$$

Strain in steel bar$=\frac{P}{2}\div E_S$, where E_S=modulus of elasticity for steel.

Strain in brass bar$=\frac{10-P}{1\cdot5}\div E_B$, where E_B=modulus of elasticity for brass.

The strains are equal, hence

$$\frac{\frac{P}{2}}{E_S}=\frac{\frac{10-P}{1\cdot5}}{E_B} \text{ and } \frac{P}{2E_S}=\frac{10-P}{1\cdot5E_B}.$$

Substituting the values of the moduli of elasticity,

$$\frac{P}{2\times30,000,000}=\frac{10-P}{1\cdot5\times10,000,000}$$

$$\frac{P}{60,000,000}=\frac{10-P}{15,000,000}.$$

Cross multiplying, 15,000,000 P=60,000,000 (10−P).
Dividing through by 15,000,000,

$$P=4\ (10-P)=40-4P.$$

$$P+4P=40, \text{ or } 5P=40, \text{ from which } P=\underline{8 \text{ tons.}}$$

Therefore load on steel bar, $P=\underline{8 \text{ tons.}}$

Load on brass bar$=10-P=10-8=\underline{2 \text{ tons.}}$

The student may of course, if he prefer, use the formulæ deduced in connection with compound bars.

Thus,

$$P=\frac{F}{1+\frac{A_B E_B}{A_S E_S}},$$

where F=load on the combined bar,
P=load taken by steel bar,
A_S=sectional area of steel bar,
A_B= ,, ,, brass ,,
E_S=modulus of elasticity of steel,
E_B= ,, ,, brass.

Substituting the known values, we get

$$P=\frac{10}{1+\frac{2\times\frac{3}{4}\times10,000,000}{2\times1\times30,000,000}}=\frac{10}{1+\frac{1}{4}}$$

$$=\frac{10}{1\cdot25}=\underline{8 \text{ tons.}}$$

Let $\quad Q$=the load taken by the brass bar.

Then $\quad Q=\dfrac{F}{1+\dfrac{A_S E_S}{A_B E_B}}=\dfrac{10}{1+\dfrac{2\times 1\times 30{,}000{,}000}{2\times\frac{3}{4}\times 10{,}000{,}000}}$

$$=\frac{10}{1+\frac{3}{\frac{3}{4}}}=\frac{10}{5}=\underline{2 \text{ tons.}}$$

Q may of course be obtained by subtraction when P has been calculated, and vice versa. Thus, if P=8 tons, and the total load F on the bar is 10 tons, it follows that Q will be 10−8=2 tons.

Having found the respective loads taken by the bars, it is a simple matter to find the stress.

$$\text{Stress in steel bar}=\frac{P}{A_S}=\frac{8}{2\times 1}=\underline{4 \text{ tons per square inch.}}$$

$$\text{Stress in brass bar}=\frac{Q}{A_B}=\frac{2}{2\times\frac{3}{4}}=\underline{1{\cdot}33 \text{ tons per square inch.}}$$

65.—**Work done in Stretching : Resilience.** (5) A load of 100 lbs. is gradually applied to a steel wire 50 feet long and ·075 inch diameter. What is the work done in stretching the wire? E=30,000,000 lbs. per square inch. If the elastic limit of the steel is 35,000 lbs. per square inch, what is the resilience of the wire?

The work done must not be confused with the resilience. The latter is the work done in stretching *up to the elastic limit of the material.* Hence the reason for giving the numerical value of the limit in the second part of the question.

Work done in stretching wire=mean resistance × space overcome.

The mean resistance is of course the load on the wire divided by 2, or $\frac{0+100}{2}$=50 lbs., the load being nil to commence with. The space overcome is the amount the wire is extended, and this can be calculated from the particulars given. The first thing to do is to find the stress in the wire, after which the extension can be calculated.

$$\text{Stress}=\frac{\text{load}}{\text{sectional area}}=\frac{100}{{\cdot}075^2\times{\cdot}785}=\underline{22{,}600 \text{ lbs. per square inch.}}$$

The modulus of elasticity is given as 30,000,000 lbs. per square inch, which means that a load of 30,000,000 lbs. would double the length of a bar of steel 1 square inch sectional area, assuming the bar would not break before this load was put upon it. We have just seen that a wire ·075 inch diameter loaded with

100 lbs. has a load of 22,600 lbs. to the square inch. The question then is this:—If 30,000,000 lbs. will produce an extension of 50 feet or 600 inches in a wire 1 square inch sectional area, what extension will a load of 22,600 lbs. produce? Stating this as a simple proportion, we have

$$30{,}000{,}000 : 600 :: 22{,}600 : x$$

$$x = \frac{600 \times 22{,}600}{30{,}000{,}000} = \cdot 452 \text{ inch.}$$

(We may also find the extension by using the relation $\frac{x}{l} = \frac{f}{E}$.)

The wire would therefore be extended by ·452 inch.

$\therefore$ Work done in stretching wire $= 50 \times \cdot 452 = \underline{22 \cdot 6 \text{ inch lbs.}}$

Resilience = work done in stretching up to elastic limit.

The elastic limit is 35,000 lbs. per square inch. The load required on the wire to produce this stress would be $35{,}000 \times \cdot 075^2 \times \cdot 785 = 154 \cdot 5$ lbs. To find the extension due to this load (or to the stress of 35,000 lbs. per square inch), we have the proportion

$$30{,}000{,}000 : 600 :: 35{,}000 : x$$

$$x = \frac{600 \times 35{,}000}{30{,}000{,}000} = \cdot 7 \text{ inch.}$$

Using the relation,

Work done = mean resistance × space overcome,

then,

Work done in stretching wire up to the elastic limit

$$= \frac{0 + 154 \cdot 5}{2} \times \cdot 7 = \underline{54 \cdot 1 \text{ inch lbs.}}$$

It has been shown that

$$\text{Resilience} = \frac{f^2}{E} \times \tfrac{1}{2} \text{ volume of bar,}$$

so we may use this to check the result just obtained. In this relation, f is the elastic limit of the material, i.e. 35,000 lbs. per square inch.

Substituting the known values,

$$\text{Resilience} = \frac{35{,}000^2}{30{,}000{,}000} \times \tfrac{1}{2} \times \cdot 075^2 \times \cdot 785 \times 50 \times 12.$$

$$= \underline{54 \cdot 1 \text{ inch lbs.}}$$

$$\left.\begin{array}{r} \text{Work done} = \underline{22 \cdot 6 \text{ inch lbs.}} \\ \text{Resilience} = \underline{54 \cdot 1 \text{ inch lbs.}} \end{array}\right\} \text{ANSWERS.}$$

(6) Referring to the previous question, what would be the work done in stretching the *wire if the load were applied suddenly instead of gradually?*

We have shown that the work done by a suddenly applied load in stretching a bar is four times that done by a gradually applied load of the same amount. The work done by the gradually applied load was found to be 22·6 inch lbs. Therefore, the work done if the load be applied suddenly will be $22 \cdot 6 \times 4 = \underline{90 \cdot 4}$ <u>inch lbs.</u>

66.—**Stresses due to Dead and Live Loads together.** (7) A wrought-iron bar is required to carry a dead load of 15 tons and a live load of 10 tons. Taking the ultimate stress of wrought iron to be 22 tons per square inch, and assuming a factor of safety of 5, what should be the diameter of the bar?

$$\text{Safe working stress} = \frac{\text{ultimate stress}}{\text{factor of safety}} = \frac{22}{5} = 4 \cdot 4 \text{ tons per square inch.}$$

The total load is equal to $15+10=25$ tons, of which 10 tons is applied suddenly. Now we have shown that a load suddenly applied produces a stress double that which would be caused by the same load applied gradually. The live load of 10 is consequently equivalent to a dead load of 20 tons.

The bar must therefore be capable of carrying a total dead load of $15+20=35$ tons.

If f = the safe working stress in tons per square inch,
F = the total dead load in tons,
A = area of section in square inches,

then $f=\frac{F}{A}$, from which $A=\frac{F}{f}$.

In the present problem, F is 35 tons and f 4·4 tons per square inch.

Hence $A=\frac{35}{4 \cdot 4}=7 \cdot 95$ square inches.

The diameter of the bar is found from the relation, $\frac{\pi}{4}d^2=A$, where d = diameter.

$$\frac{\pi}{4}d^2=A, \text{ and } d^2=\frac{A}{\frac{\pi}{4}}=\frac{4A}{\pi}.$$

Then $d=\sqrt{\frac{4A}{\pi}}=2\sqrt{\frac{A}{\pi}}$. Substituting the known value of A,

$$d=2\sqrt{\frac{7 \cdot 95}{\pi}}=2\sqrt{2 \cdot 53}=2\times 1 \cdot 59=\underline{3 \cdot 18 \text{ inches.}}$$

∴ Diameter of bar required = <u>3·18 inches.</u>

A bar $3\frac{1}{4}$ inches diameter would probably be used.

67.—**Stresses due to Impulsive Loads.** (8) A steel rod 8 feet 4 inches long and 2 square inches sectional area is provided with a collar at one end. The rod is suspended with the collar lowermost, and a load of 1000 lbs. is then dropped upon the collar from a point one inch above. Taking E for steel to be 30,000,000 lbs. per square inch, find the stress produced in the rod.

The effect of the falling weight is to stretch the bar and so do work upon it. The work given out by the falling weight must obviously be equal to the work done in stretching.

The former is 1000 $(1+x)$ inch lbs., where x is the extension produced in inch units, and the latter is $\frac{f^2}{E}\times\frac{1}{2}$ volume of rod, where f =stress produced in lbs. per square inch, which must be assumed not greater than the elastic limit of the material.

We have then the equation,

$$1000\,(1+x)=\frac{f^2}{E}\times\frac{1}{2}\text{ volume of rod.}$$

Neglecting x, which will be, comparatively speaking, very small, the equation becomes

$$1000=\frac{f^2}{E}\times\frac{1}{2}\text{ volume of rod, from which}$$

$$f^2=\frac{1000\times E}{\frac{1}{2}\text{ volume of rod}}\text{ and }f=\sqrt{\frac{1000\times E}{\frac{1}{2}\text{ volume of rod}}}.$$

Substituting the known values

$$f=\sqrt{\frac{1000\times 30{,}000{,}000}{\cdot 5\times 2\times 100}}=\sqrt{300{,}000{,}000},$$

$$\therefore\ f=\underline{17{,}300\text{ lbs. per square inch.}}$$

The same result is obtained by using the formula

$$f=\sqrt{\frac{2FhE}{Al}},$$

where F=load in lbs.,
h=height through which F falls in inch units,
A=sectional area of rod in square inches,
l=length of rod in inches,
f=stress produced in lbs. per square inch.

Substituting the known values

$$f=\sqrt{\frac{2\times 1000\times 1\times 30{,}000{,}000}{2\times 100}}$$

$$=\sqrt{300{,}000{,}000}=17{,}300\text{ lbs. per square inch.}$$

$\therefore$ Stress produced in rod=$\underline{17{,}300\text{ lbs. per square inch.}}$

It will be interesting now to consider how the result is affected if we do not neglect the factor x, the extension produced in the rod.

The original equation is

$$1000\ (1+x)=\frac{f^2}{E}\times\frac{1}{2}\text{ volume of rod.}$$

Now $x=\frac{lf}{E}$, where l=length of rod=100 inches.

Substituting for x in the equation, we obtain

$$1000\left(1+\frac{100f}{30{,}000{,}000}\right)=\frac{f^2}{30{,}000{,}000}\times\frac{2\times100}{2}$$

$$1000+\frac{100{,}000}{30{,}000{,}000}f=\frac{f^2\times100}{30{,}000{,}000}$$

$$1000+\frac{1}{300}f=\frac{f^2}{300{,}000}.$$

Multiplying throughout by 300,000 and rearranging the terms,

$$f^2-\frac{300{,}000}{300}f=1000\times300{,}000$$

$$f^2-1000f=300{,}000{,}000.$$

Completing the square, $f^2-1000f+500^2=300{,}000{,}000+250{,}000.$

Extracting the square root, $f-500=\pm\sqrt{300{,}250{,}000}$

f=500+17,330=<u>17,830 lbs. per square inch.</u>

Or, using the formula which takes x into account,

$$f=\frac{F}{A}+\frac{\sqrt{2FhEAl+F^2l^2}}{Al}$$

$$=\frac{1000}{2}+\frac{\sqrt{2\times1000\times1\times30{,}000{,}000\times2\times100+1000^2\times100^2}}{2\times100}$$

$$=500+\frac{\sqrt{12{,}000{,}000{,}000{,}000+10{,}000{,}000{,}000}}{200}$$

$$=500+\frac{\sqrt{12{,}010{,}000{,}000{,}000}}{200}$$

$$=500+\frac{3{,}466{,}000}{200}=500+17{,}330=\underline{17{,}830\text{ lbs. per square inch.}}$$

$\therefore$ Real stress=<u>17,830 lbs. per square inch.</u>

It will be observed that if the extension x be not neglected in the general equation, the stress appears to be greater than is the case if it be neglected. The difference, however, is comparatively slight, viz. only 530 lbs. in over 17,000 lbs., so that it is sufficient in most cases to neglect x, and thus avoid the necessity of solving a quadratic equation or employing a clumsy formula.

(9) It is found that a steel rod is extended $\frac{1}{4}$ inch by a load of 1200 lbs. applied gradually. From what height would a load of 50 lbs. have to be dropped on to a collar formed on the lower end of the rod to produce the same stress as that caused by the load of 1200 lbs. gradually applied?

This example may be very simply solved without having to calculate the stress. Thus, we know that the work done by the gradually applied load in stretching the rod is equal to the work given out by the falling weight, and both these quantities of work may be calculated from the particulars given.

Let F = the gradually applied load.
,, x = the extension.
,, w = the weight of the falling load.
,, h = height through which the load falls.

Then the work done by F in extending the rod an amount x will be $\frac{1}{2}Fx$, and the work given out by the falling load, w, will be $w(h+x)$. These two are equal, and consequently we have the equation

$$\frac{1}{2}Fx = w(h+x)$$

from which,

$$\frac{1}{2}Fx = wh + wx.$$

$$wh = \frac{1}{2}Fx - wx.$$

$$\therefore\ h = \frac{x(\frac{1}{2}F - w)}{w}.$$

Substituting the known values

$$h = \frac{\frac{1}{4}(\frac{1}{2}\times 1200 - 50)}{50} = \frac{\frac{1}{4}(600-50)}{50} = \frac{550}{200} = \underline{\underline{2{\cdot}75 \text{ inches.}}}$$

$\therefore$ Height through which the 50 lbs. would have to fall = $\underline{\underline{2{\cdot}75 \text{ inches.}}}$

(10) An iron rod is 20 feet long and $1\frac{1}{2}$ inches diameter. A weight of 40 lbs. is allowed to fall from a height of 12 feet on to a solid collar formed on the lower end of the rod. Find the stress, the extension, and the strain produced. E = 25,000,000 lbs. per square inch.

The first part of this question is similar to the previous one, but in addition to the stress we have also to find the strain.

To find the stress we may make use of the formula

$$f = \sqrt{\frac{2FhE}{Al}},$$

where f = stress in lbs. per square inch,
h = height through which the weight F is dropped (inches),
A = sectional area of bar in square inches,
l = length of bar in inches.

Now, $A = 1{\cdot}5^2 \times {\cdot}785 = 1{\cdot}77$ square inches.

Substituting the data given,

$$f = \sqrt{\frac{2 \times 40 \times 12 \times 12 \times 25{,}000{,}000}{1{\cdot}77 \times 20 \times 12}}$$

$$= \sqrt{\frac{48 \times 25{,}000{,}000}{1{\cdot}77}} = \sqrt{678{,}000{,}000}$$

$$= \underline{26{,}050 \text{ lbs.}} \text{ per square inch.}$$

Hence, Stress produced $= \underline{26{,}050 \text{ lbs.}}$ per square inch.

Having found the stress, it is a simple matter to find the extension. Thus,

$$\text{Extension, } x = \frac{lf}{E} = \frac{20 \times 12 \times 26{,}050}{25{,}000{,}000} = \underline{{\cdot}25 \text{ inch.}}$$

$$\text{Strain} = \frac{\text{extension}}{\text{original length}} = \frac{{\cdot}25}{20 \times 12} = \underline{{\cdot}00104.}$$

The strain might be found direct from the relation,

$$\frac{\text{Stress}}{\text{Strain}} = \text{Modulus of elasticity.}$$

Thus, $$\frac{26{,}050}{\text{Strain}} = 25{,}000{,}000$$

$$\text{Strain} = \frac{26{,}050}{25{,}000{,}000} = \underline{{\cdot}00104.}$$

We have then,

$$\left.\begin{array}{r} \text{Stress} = \underline{26{,}050 \text{ lbs. per square inch.}} \\ \text{Extension} = \underline{{\cdot}25 \text{ inch.}} \\ \text{Strain} = \underline{{\cdot}00104.} \end{array}\right\} \text{ANSWERS.}$$

68.—**Stresses due to the forces of Expansion and Contraction.** (11) It is required to straighten the walls of a building which have become bulged. For this purpose an iron bolt, 2 inches diameter and 20 feet long, is heated to a temperature of 500 degrees F., when the nuts are screwed up against large washers made to bear against the walls. Find the pull exerted on the walls when the bar has been allowed to cool down to a temperature of 300 degrees F.

Coefficient of expansion of iron, ·0000066; E=25,000,000 lbs. per square inch.

It is first necessary to find the amount of contraction which takes place on cooling.

Range of temperature=(500—300)=200 degrees F.

The coefficient of expansion of iron is given as ·0000066, so that the bolt will contract ·0000066 of its length for every degree through which it is cooled.

Total contraction $=20\times12\times{\cdot}0000066\times200={\cdot}317$ inch.

Now a load of 25,000,000 lbs. would extend or compress an iron bar 1 square inch in sectional area and 20 feet long by 20 feet (on the usual assumptions). What force then would be required to compress a rod 1 square inch in sectional area and 20 feet long ·317 inch?

$$25{,}000{,}000 : 20\times12 :: x : {\cdot}317.$$

$$x=\frac{25{,}000{,}000\times{\cdot}317}{240}=33{,}000 \text{ lbs. (per square inch).}$$

The bolts are 2 inches diameter and have a sectional area of $2^2\times{\cdot}785=3{\cdot}14$ square inches. The force required per square inch of section is 33,000 lbs., so that the total force required will be $33{,}000\times3{\cdot}14=103{,}600$ lbs. This will be the force on the walls.

$\therefore$ Force on walls = 103,600 lbs.

We showed at the end of the chapter that the stress produced in a bar which is prevented from expanding or contracting freely is

$$f=arE,$$

where a = coefficient of expansion,

r = range of temperature through which the bar is heated or cooled.

Applying this to the problem under consideration, we get

$f={\cdot}0000066(500-300)25{,}000{,}000=33{,}000$ lbs. per square inch,

which agrees with the result already obtained. The pull on the walls is obtained by multiplying this stress by the sectional area of the bolts. Note that the length of the bolt need not be given if we use the formula, since it cancels out, as will be seen on referring to the text.

(12) A steel Lancashire boiler, 8 feet diameter by 30 feet long, is uniformly heated from a temperature of 50 degrees F., the temperature of the feed water, to 350 degrees F., the latter temperature corresponding to the steam pressure. How much does it expand for this rise of temperature? If the boiler be suddenly cooled down to 150 degrees F., find the stresses set up in the plates.

Coefficient of expansion for steel, ·00000672; E=30,000,000 lbs. per square inch.

Expansion = length × coefficient of expansion × rise of temperature.

$={30\times12\times{\cdot}00000672\times(350-50)}={\cdot}726$ inch.

$\therefore$ Expansion = ·726 inch.

If the boiler be suddenly cooled, the effect is to a large extent the same as though the whole structure were held rigidly at each end so that contraction could not take place, with the result that injurious stresses are set up in the plates. In cooling down to 150 degrees F., the boiler would contract $30 \times 12 \times \cdot 00000672(350-150)$ $= \cdot 484$ inch.

A force of 30,000,000 would stretch a bar of steel 30 feet long and 1 square inch in sectional area 30 feet or 360 inches, so we have to find what force would be required to produce an extension (or a shortening) of ·484 inch.

$$30{,}000{,}000 : 360 :: x : \cdot 484$$

$$x = \frac{30{,}000{,}000 \times \cdot 484}{360} = \underline{40{,}320 \text{ lbs. per square inch.}}$$

$$= \underline{18 \text{ tons per square inch.}}$$

If we use the formula,

$$f = arE,$$

where
a = coefficient of expansion,
r = range of temperature,
E = modulus of elasticity,

then $f = \cdot 00000672(350-150)30{,}000{,}000$
$= \cdot 00000672 \times 200 \times 30{,}000{,}000$
$= \underline{40{,}320 \text{ lbs. per square inch,}}$ as found above.

Hence, Stress in plates = $\underline{18 \text{ tons per square inch.}}$

If we assume the plates to be cooled down suddenly to 50 degrees F. instead of 150, then by similar calculation we should find the stress to be almost equal to the ultimate stress of the material. The folly of rapidly cooling the hot plates of steam boilers will therefore be apparent. Note that the diameter of the boiler does not enter into the question.

Chapter V

STRENGTH OF CYLINDRICAL VESSELS EXPOSED TO FLUID PRESSURE

69.—Many vessels of cylindrical form are employed in actual practice for storing fluids under pressure, as, for example, steam boilers, air receivers, etc., the plates of which are put in a state of stress as a result of the pressure acting on them. The shells of such vessels are usually thin in comparison with their diameter, and it is a simple matter to determine the stresses set up in the plates. Before proceeding, it will be well to consider for a moment whether or not the atmospheric pressure plays any part in setting up stresses in the plates.

Every one is familiar with the fact that the pressure of the atmosphere is due to the weight of a column of air of, say, 1 square inch sectional area, and of a height equal to that of the atmosphere, the weight of this column being 15 lbs. very nearly. The pressure of the atmosphere is therefore 15 lbs. per square inch.

Now the vessel when empty will be subjected to the atmospheric pressure both internally and externally, so that the resultant effect will be nil. When the pressure, say p (measured above the atmospheric pressure), due to the steam or air, acts internally, the total internal pressure will be $p+15$, and the external pressure 15 lbs. per square inch. The effective pressure is consequently $p+15-15=p$ lbs. per square inch, so that the atmospheric pressure does not enter into the question.

Fig. 20

The pressure in a cylindrical vessel acts radially and uniformly all round the inner circumference, as indicated by the arrows of Fig. 20, and its tendency is to burst the vessel longitudinally. Before the strength of the vessel to resist bursting can be calculated, it is necessary to determine the total force which tends to cause bursting.

It will be convenient to consider the cylinder as being composed of a number of rings of unit length, i.e. 1 inch, each of which is independent of the others in sustaining pressure. Now consider one of these rings, as represented by Fig. 21. Let xy represent a very small surface of the circumference of the ring, and let this surface make an angle θ with the horizontal diameter of the ring. The total pressure acting on xy will be $p \times xy$, where p is the pressure of the fluid. This may be resolved into vertical and horizontal components. The vertical component is $p \times xy \times \cos\theta$. It is the sum of all such vertical components which constitutes the total force tending to burst the vessel along a plane represented by the horizontal centre line.

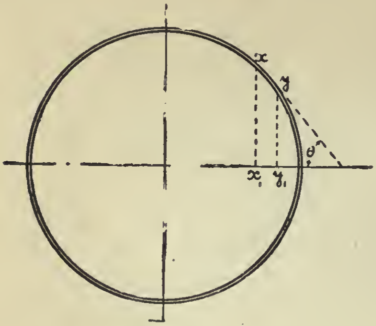

Fig. 21

Now $xy \cos\theta$ is equal to the projection x_1y_1 on the horizontal diameter, so that the vertical component will be equal to $p \times x_1y_1$. Clearly, the sum of all the vertical components of p is equal to $p \times d$.

Consequently, *the force tending to burst the vessel longitudinally is equal to the pressure per square inch multiplied by the diameter of the vessel in inches*, assuming the length of the vessel to be unity. Hence, if the vessel be l inches long, the total force tending to burst it will be equal to the above force multiplied by l. We may thus suppose the pressure to be acting in the manner indicated by Fig. 22, the general effect being to burst the vessel into two symmetrical halves along a plane passing through the centre of the vessel. It is clear that as a result of the forces tending to produce failure, the plates of which the vessel is constructed will be subjected to tensile stress.

Fig. 22

70.—Now consider any thin plain cylindrical vessel which we may suppose to be storing either steam, air, or water under pressure, and, for convenience, assume there are no end plates.

Let p=the internal pressure of the fluid, in lbs. per square inch.
,, d=diameter of vessel, in inches.
,, l=length ,, ,,
,, t=thickness of plates ,,
,, f_t=tensile stress in plates due to the pressure, in lbs. per square inch.

We have not stated whether d is the internal, the external, or the mean diameter of the vessel, but this is immaterial, since the following consideration is based on the assumption that the thickness of the plates, t, is small in comparison with the diameter of the vessel, d.

The total force tending to cause rupture will be $p \times d \times l$, and this is resisted by a force equal to $2 \times t \times l \times f_t$. (See Fig. 23.)

We have then the equation,

$$pdl = 2tlf_t, \text{ from which } f_t = \frac{pd}{2t}.$$

Note that the length l cancels out, so that the stress set up in the plates of a vessel exposed to internal pressure is independent of the length of the vessel. With regard to the end plates, it is quite obvious that these will assist the shell in resisting the tendency to rupture, and consequently near the ends the stress in the plates will be less than that indicated by the above relation. Any portion of the shell, however, situated some distance from the ends, will not be strengthened appreciably by the end plates, and as *the strength of a structure is that of its weakest part*, the relation $f_t = \frac{pd}{2t}$ is correctly used for finding the stress.

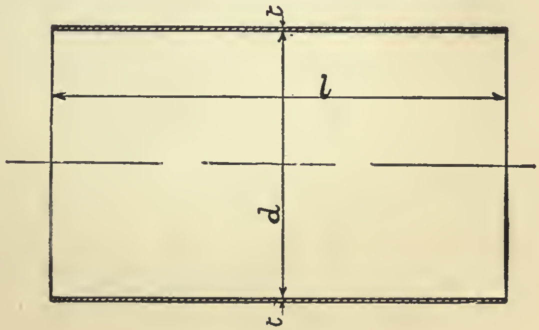

Fig. 23

A common problem in connection with cylindrical vessels exposed to internal pressure is to find the pressure at which a vessel can be safely worked, allowing a safe tensile stress, f_t, on the plates. For this purpose the foregoing relation may be rearranged thus:—

$$p = \frac{2tf_t}{d}.$$

If we let f_t^1 represent the ultimate tensile stress of the plate material, then the relation, $p = \frac{2tf_t^1}{d}$ enables us to determine the pressure at which the vessel would burst.

This simple formula is the one generally used for calculating the pressure which steam boilers, air receivers, and the like are capable of carrying, but as it assumes the plates to be solid, i.e. to have no joints, it requires modification so as to take into account the effect of the joints. We shall deal with this presently.

71.—So far, we have considered the bursting of a cylindrical vessel in a longitudinal direction only, but it is evident that there is also a tendency to burst the shell in a circumferential direction, i.e. to separate the vessel into two cylindrical portions as a result of the pressure acting on the end plates. The circumferential strength may be easily determined in the following manner:—

The total force acting on the end plates and tending to sever the vessel into two cylindrical portions is equal to $p \times \frac{\pi}{4} \times d^2$. The tendency to rupture is resisted by a circular strip of metal of thickness t, and length equal to the circumference of the shell, viz. πd. The total resisting force is then $t \times \pi d \times f_t$.

Equating the total bursting force to the resisting force, we have

$$p \frac{\pi}{4} d^2 = t \pi d f_t, \text{ from which } \underline{f_t = \frac{pd}{4t}}.$$

Rearranging to obtain the pressure, we get

$$\underline{p = \frac{4tf_t}{d}}.$$

If the student will now compare these relations with the previous ones, he will see that *the strength of a cylindrical vessel to resist internal pressure, in a circumferential direction, is just twice what it is in a longitudinal direction.* It follows that when determining the strength of a cylindrical boiler or similar vessel, the strength to resist rupture in a longitudinal direction is what chiefly concerns us.

72.—**Strength of Thin Spherical Shell.** The ordinary thin cylindrical shell has been shown to be twice as strong circumferentially as longitudinally. Consider next a thin shell of spherical form, of diameter d, subjected to an internal pressure p. The effect of the pressure is to burst the shell into two equal halves, and the vessel is just as liable to burst along any one section through the middle as along any other section, providing the plates be of uniform strength and thickness throughout. The total force tending to cause rupture is $p \times \frac{\pi}{4} \times d^2$, and this is resisted by a circular strip of metal of thickness t and length πd.

We have then the same equation as we obtained for the strength of a cylindrical shell in a circumferential direction, viz.:—

$$p \times \frac{\pi}{4} \times d^2 = t \times \pi \times d \times f_t, \text{ from which } \underline{f_t = \frac{pd}{4t}}.$$

A spherical shell, therefore, being of uniform strength throughout, is twice as strong to resist internal pressure as a cylindrical shell of the same diameter and thickness of plate. For this reason, from the strength point of view, a spherical boiler is much superior to a cylindrical boiler, but the spherical form is not convenient for boiler purposes, and hence the plain cylindrical form is the one mostly adopted in boiler construction.

73.—**Strength of Thin Shell of Oval Form.** Although the oval form of shell is an unusual one, it is interesting to consider its strength to resist internal pressure. It may be mentioned that a tube of oval section is used in the well-known Bourdon pressure gauge, the gauge pointer being moved by the opening out of a spiral tube of such section, caused by the admission into the tube of steam or other fluid from the vessel to which the gauge is attached.

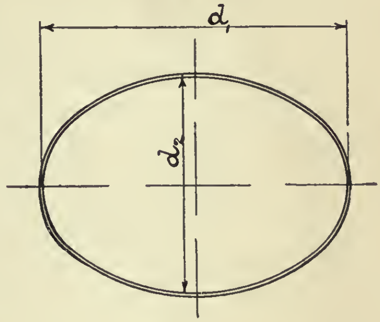

Fig. 24

Fig. 24 represents a thin oval shell supposed to be under internal pressure. Consider first the stress in the shell at the parts cut by taking a horizontal section through the middle.

If we let d_1 equal the greater diameter, and p the pressure, the total force tending to burst the vessel into two equal halves along the horizontal centre will be $p \times d_1 \times l$. This tendency to burst is resisted by two strips of metal of length l and thickness t, the total resisting force being $2 \times t \times l \times f_{t1}$, where f_{t1} equals the tensile stress in the material at the horizontal centre.

Therefore, $p \times d_1 \times l = 2 \times t \times l \times f_{t1}$, from which $f_{t1} = \dfrac{pd_1}{2t}$.

Similarly, if we consider the stress in the shell at the parts cut through by taking a vertical section, the diameter there being represented by d_2, we find this to be $f_{t2} = \dfrac{pd_2}{2t}$.

The stress in a thin oval shell due to internal pressure is thus greatest at places of maximum curvature or smallest radius, and least at places of minimum curvature or greatest radius.

It must be clearly borne in mind that the foregoing results are only correct when the thickness of the vessel is small in comparison with the diameter. They could not, for instance, be safely applied to determine the true strength of a small cast-iron pipe or of a hydraulic cylinder. The determination of the strength of such vessels requires a more advanced knowledge than the student yet possesses, and must consequently be left alone for the present.

74.—Cylindrical Vessels subjected to External Fluid Pressure. In steam boiler work, although the outer shell is usually a cylindrical vessel subjected to internal pressure, there are frequently internal cylinders or tubes exposed to external pressure. In the well-known Lancashire boiler, for instance, there are two cylindrical tubes or flues, of rather more than one-third the diameter of the outer shell, running from one to the other end of the boiler. The water, under the same pressure as that of the steam, surrounds each of these tubes, which, in consequence, instead of being exposed to a bursting action, as in the case of the shell, are exposed to a crushing or collapsing action.

Whilst, therefore, the shell must be sufficiently strong to resist bursting, the internal tubes must be capable of resisting collapse, and it is necessary for us to consider briefly how the strength of such tubes is determined.

In dealing with shells exposed to internal pressure we had no difficulty in deducing from first principles a formula connecting together the pressure, the diameter of the shell, the thickness of, and the stress in, the plates.

Unfortunately, we cannot from first principles deduce satisfactorily a formula for cylindrical vessels exposed to external pressure, and we are bound to fall back upon the results of experiments for true information respecting the strength of these vessels. From the results of such experiments, a number of rules or formulæ have been devised, and in actual practice one or other of these rules is generally employed.

We have seen that in the case of a cylindrical vessel, subjected to internal pressure, the length of the vessel did not enter into the calculation of strength; in other words, the strength of the vessel to resist bursting is independent of the length. It seems natural to suppose that if the pressure, instead of acting internally, acted externally, the length of the vessel would again have no influence on its strength. Experiments have, however, shown that such is not by any means the case.

In 1858, Sir William Fairbairn conducted a series of experiments with the object of determining the strength of cylindrical tubes subjected to external pressure, and from the results of his experiments he deduced the following rule:—

$$p=806{,}300\,\frac{t^{2\cdot19}}{ld},$$

where p=the pressure required to cause collapse of the tube, in lbs. per square inch,
t=thickness of plates, in inches,
l=length of tube, in feet,
d=diameter of tube, in inches.

From this it is seen that the pressure required to cause failure of a tube by collapse varies as the 2·19th power of the plate thickness, and inversely as the length and the diameter of the tube.

Thus, the longer the tube the weaker it is to resist collapse due to external pressure.

When a cylindrical vessel is under internal pressure the tendency of the pressure is to maintain the vessel in a truly circular form. Thus, if the vessel were not truly cylindrical to commence with, the pressure, acting uniformly and radially all over the internal surface, would tend to remedy the defect. On the other hand, should a vessel, slightly out of the true cylindrical form, be exposed to external pressure, the tendency of the pressure is to aggravate the imperfection, and the experiments carried out by Fairbairn showed that a slight deviation from the true circle very materially reduced the strength of the vessel to resist collapse. In one experiment, two tubes 37 inches long, 9 inches diameter, and about $\frac{1}{8}$ of an inch thick were tested, one of the tubes having a lap joint, which causes it to depart slightly from the true circular form, and the other a butt joint, which retains the true form. The former tube collapsed with a pressure of 262 lbs. per square inch, whilst the latter withstood a pressure of 378 lbs. per square inch before collapsing.

Thus, a very slight departure from the true circular form of the tube resulted in a reduction of strength of approximately one-third.

This illustrates the importance of making cylindrical vessels or tubes subject to external pressure truly circular, and in modern boiler practice special attention is given to this point. The tubes of Lancashire and other boilers nowadays are made with welded joints which do not appreciably alter the circularity, as do the lap joints which were at one time largely used. Long lengths of plain tube are also avoided in modern boiler practice, the tubes usually being constructed of a number of short lengths connected together by flanged seams, which materially increase the strength of the tubes, as a whole, to resist collapse. The tubes are also frequently made of a corrugated form, which is both stronger than the plain form and better able to accommodate the expansive movements which are set up under working conditions.

The above rule, generally known as Fairbairn's rule, has been much used in the past for calculating the strength of long circular flue tubes, but experience seems to show that the rule gives too high pressures for short tubes and too low for long ones. It is quite inapplicable to the flue tubes of boilers strengthened by flanged seams. For this reason, other rules have been devised. One of these is Longridge's rule, which gives not the collapsing, but the safe working pressure of long, plain iron tubes under external pressure :—

$$\text{Working pressure}=\frac{50t^2}{d\sqrt{l}}-\frac{d}{l},$$

where t=thickness of plates, in thirty-seconds of an inch,
d=diameter of tube, in inches,
l=length, in feet.

Another rule, viz. Seaton's, may be given. This is,

$$t=\sqrt{\frac{pld}{C}}+2,$$

where t=thickness of plates, in thirty-seconds of an inch, required for a working pressure of p lbs. per square inch,
l=length of tube, in inches,
d=diameter ,, ,,
C=a constant=900 for iron tubes and 1000 for steel.

The rule makes due allowance for uniform wear of the surface of the tube due to corrosion.

The Board of Trade, in their "Instructions as to the Survey of Passenger Steamships," give the following rule in connection with circular steel furnaces, the longitudinal seams of which are welded, or made with single butt straps double riveted, or double butt straps single riveted :—

$$p=\frac{99{,}000\times t^2}{(l+1)\times d},$$

where p=working pressure, in lbs. per square inch,
t=thickness of plates, in inches,
l=length of furnace, in feet,
d=diameter of furnace, in inches.

This rule, it should be explained, does not apply if the working pressure be found to exceed that given by the formula,

$$p=\frac{9{,}900\times t}{d}.$$

The latter formula limits the crushing stress on the material to 4950 lbs. per square inch.

The furnace tubes of Lancashire and other boilers, as previously explained, are generally constructed of a number of short rings welded longitudinally, the ends of each ring being flanged and riveted to the next, with a caulking ring intervening, and for such tubes the Board of Trade give the following formula for determining the working pressure, providing the length of each ring, measured over the flanges, does not exceed ($120t-12$), and that certain conditions in regard to the workmanship are complied with :—

$$p=\frac{9900\times t}{3\times d}\left(5-\frac{l+12}{60\times t}\right),$$

where t=thickness of plates, in inches,
d=diameter of tube, in inches,
l=length of each ring, in inches.

75.—**Strength of Flat Plates.** In most types of steam boiler, in addition to the cylindrical plates, there are frequently plates of flat form exposed to the pressure. Now flat plates are comparatively weak to sustain pressure, and in most cases it is necessary to strengthen them by means of stays of one form or another.

Unfortunately, the theoretical investigation of the strength of a flat plate under pressure is difficult and unsatisfactory, and in actual boiler design all flat surfaces, unless of small area, are efficiently stayed, and the pressure which may then be safely carried, or the thickness of plate required for a certain pressure, is determined by means of certain rules.

Thus the Board of Trade rule for the flat surfaces of marine boilers is as follows :—

$$p=\frac{C(t+1)^2}{S-6},$$

where p=allowable working pressure, in lbs. per square inch,
t=thickness of plate, in sixteenths of an inch,
S=surface of plate supported by one stay, in square inches,
C=a constant, the value of which depends on whether or not the plates are exposed to impact of heat, the form of the stays, etc.

As regards the constant C, the value of this may be as little as 36, or as much as 240. The lower value applies when the plates are wrought iron and exposed to the impact of heat, with steam in contact with them, and when the stays are screwed into the plates, their ends being riveted over to form substantial heads. The higher value applies when the plates are of steel not exposed to impact of heat or flame, when the stays are fitted with nuts on both sides of the plates, and when doubling strips, not less in width than two-thirds the pitch of the stays and of the same thickness as the plates, are securely riveted to the outside of the plates they cover.

If the pressure be known, and the required thickness of plate is to be determined, the formula may be conveniently rearranged. thus :—

$$t=\sqrt{\frac{p(S-6)}{C}}-1.$$

The usual kinds of stay adopted in boiler work at the present time are those known as gusset stays, longitudinal bolt stays, and screwed stays.

The flat end plates of Lancashire boilers, for example, are strengthened by gusset stays, which consist of flat plates riveted between angle irons, two of which are riveted to the end plate, and two to the shell. The thrust on the end plate, or at least a portion of it, is thus transmitted through the gusset plate to the shell. Of course, in a Lancashire boiler, the end plates are efficiently stayed by the flue tubes, which run from end to end of the boiler, but even with such large stays a comparatively large amount of unstayed flat surface is left above the tubes (and a smaller amount below), and the gusset stays are consequently required as well.

In addition to the gusset stays it has been customary in the past to strengthen the end plates of Lancashire boilers by means of longitudinal bolt stays passed from one end of the boiler to the other, the bolts being fitted with double nuts and washers at each end, but such stays are seldom used in new boilers nowadays, because, owing to the manner in which a long bolt sags, it is questionable if they really strengthen the end plates.

In order to obviate the necessity of fitting gusset stays, the end plates of Lancashire boilers are nowadays frequently made of dished form instead of flat, a dished plate being considerably stronger than a flat plate for resisting pressure. It will be understood from what has been said in the early part of the present chapter, that if the ends of a boiler were made of hemispherical form they would be twice as strong as the boiler shell to resist bursting; if made flat they would be weaker than the shell, so a compromise is made by making them dished. If the ends be dished to a radius equal to the diameter of the shell, their strength, neglecting the weakening which results from flanging for attachment to the shell, may be supposed equal to that of the shell, because they are then portions of a sphere whose diameter is double that of the boiler shell. The end plates of water-tube boilers of the Babcock and Wilcox and the Stirling types are generally dished and unstayed.

Screwed stays are employed largely for staying the flat sides of the combustion chamber of marine boilers and the flat sides of the firebox and outer casing of locomotive type boilers. Such stays consist simply of a round bar of steel, iron, or copper, screwed along the length (or in some cases only at the ends), so that they can be screwed through the two plates which are to be secured together, after which the ends can be riveted over. They are usually spaced uniformly over the surface to be stayed, each stay supporting a certain area of plate, and consequently a certain load.

If the stays be spaced a distance s apart, then, according to Professor Unwin, the greatest stress f in the plates, due to a pressure p, is given by the relation

$$f=\frac{2s^2p}{9t^2},$$

where t is the thickness of the plates.
From this we get

$$t=\frac{s}{3}\sqrt{\frac{2p}{f}}$$

and $$s=3t\sqrt{\frac{f}{2p}}.$$

Worked Examples

76.—**Cylindrical Shells.** (1) A cylindrical steel shell 6 feet diameter and 15 feet long is required to sustain an internal pressure of 80 lbs. per square inch. Assuming the plates to be solid throughout, i.e. to have no joints, what should be the thickness of the plates?

The strength of a thin cylindrical shell to resist internal pressure is given by the relation,

$$f_t=\frac{pd}{2t},$$

where f_t=allowable tensile stress in plate material, in lbs. per square inch,
p=safe pressure, in lbs. per square inch,
d=diameter of shell, in inches,
t=thickness of plates, in inches.

We require to find t, the thickness of the plates.
Rearranging the equation,

$$t=\frac{pd}{2f_t}.$$

For safe working, f_t for steel plates may be taken to be from 10,000 to 12,000 lbs. per square inch. Taking the former figure, and substituting the given data,

$$t=\frac{80\times6\times12}{2\times10{,}000}=\cdot288 \text{ inch.}$$

The required thickness of plate is therefore ·288 inch. In all probability the plates would be made at least $\frac{5}{16}$ inch thick, or more, to allow for wasting, etc. Note that the length of the shell does not enter into the calculation.

(2) If, in the previous question, the shell were made of spherical form, what thickness of plate would be sufficient ?

We have shown in the early part of the chapter that a spherical shell is just twice as strong to resist internal pressure as a plain cylindrical shell. As the strength of the shell varies directly as the thickness, it follows that if the vessel be made of spherical form, the plates need only be one-half the thickness required for the cylindrical form.

The required thickness is then $\frac{1}{2}$ of $\cdot 288 = \underline{\cdot 144 \text{ inch.}}$

The strength of the spherical shell may of course also be determined from the relation,

$$f_t = \frac{pd}{4t}.$$

From this,

$$t = \frac{pd}{4f_t},$$

and substituting the values, $t = \dfrac{80 \times 6 \times 12}{4 \times 10{,}000} = \underline{\cdot 144 \text{ inch,}}$

which agrees with the result already found.

(3) Find the maximum and minimum stresses in an oval shell subjected to an internal pressure of 100 lbs. per square inch. The greater and lesser diameters of the shell are 3 feet and 2 feet respectively, and the thickness of the plates $\frac{3}{8}$ inch.

It has been shown that the stresses in an oval shell due to internal pressure are $f_{t1} = \dfrac{pd_1}{2t}$ and $f_{t2} = \dfrac{pd_2}{2t}$,

where f_{t1} = maximum stress, in lbs. per square inch,
f_{t2} = minimum ,, ,, ,,
p = pressure, in lbs. per square inch,
d_1 = greater diameter of shell, in inches,
d_2 = lesser ,, ,, ,,
t = thickness of plates, in inches.

Substituting the given values,

$$f_{t1} = \frac{100 \times 3 \times 12}{2 \times \cdot 375} = \underline{4800 \text{ lbs. per square inch.}}$$

This stress occurs at the parts cut by a plane passing through the centre of the shell where the diameter is a maximum.

$$f_{t2} = \frac{100 \times 2 \times 12}{2 \times \cdot 375} = \underline{3200 \text{ lbs. per square inch.}}$$

This stress occurs at the parts cut by a plane passing through the centre of the shell where the diameter is a minimum.

Maximum stress = $\underline{4800 \text{ lbs. per square inch.}}$

Minimum stress = $\underline{3200 \text{ lbs. per square inch.}}$

(4) A plain iron furnace tube 2 feet 8 inches diameter and 6 feet 10 inches long is found to collapse under a pressure of 430 lbs. per square inch. Determine the thickness of plate, using Fairbairn's rule.

According to Fairbairn's rule, the collapsing pressure

$$p=806{,}300\frac{t^{2\cdot19}}{ld},$$

where t=thickness of plates, in inches,
l=length of tube, in feet,
d=diameter of tube, in inches.

As we require to find t, it will be convenient to rearrange the equation.

Thus, $$t^{2\cdot19}=\frac{pld}{806{,}300},$$

from which $$t=\left(\frac{pld}{806{,}300}\right)^{\frac{1}{2\cdot19}}.$$

Substituting the given data,

$$t=\left(\frac{430\times6\cdot83\times32}{806{,}300}\right)^{\frac{1}{2\cdot19}}=(\cdot1167)^{\frac{1}{2\cdot19}}=\cdot1167^{\cdot457}=\underline{\cdot375\text{ inch.}}$$

Thickness of plate $=\underline{\cdot375\text{ or }\tfrac{3}{8}\text{ inch.}}$

(5) Find the pressure at which a steel furnace tube 3 feet 2 inches diameter and 30 feet long may be worked, the tube being made up of twelve separate rings, ½-inch thick; the ends of each ring are flanged and riveted to the next, and each ring is welded longitudinally.

We may conveniently apply the Board of Trade rule given in the text to this example.

$$p=\frac{9900\times t}{3\times d}\left(5-\frac{l+12}{60\times t}\right),$$

where p=working pressure, in lbs. per square inch,
t=thickness of plate, in inches,
d=diameter of tube, in inches,
l=length of each ring, in inches.

As there are twelve rings to a length of 30 feet, each ring will be 2 feet 6 inches or 30 inches long. We may neglect the space taken up by the caulking rings.

Substituting the given values,

$$p=\frac{9900\times\cdot5}{3\times38}\left(5-\frac{30+12}{60\times\cdot5}\right)$$

$$=\frac{4950}{114}\left(5-\frac{42}{30}\right)=\underline{156\cdot2\text{ lbs. per square inch.}}$$

Working pressure $=\underline{156\text{ lbs. per square inch.}}$

(6) Find by the Board of Trade rule the safe working pressure for the flat steel plates of the combustion chamber of a marine boiler, the plates being $\frac{9}{16}$-inch thick and the pitch of the stays being $7\frac{3}{4}$ inches, both horizontally and vertically. Assume the constant C to be 100.

The rule referred to is,

$$p=\frac{C(t+1)^2}{S-6},$$

where p=working pressure, in lbs. per square inch,
t=thickness of plates, in sixteenths of an inch,
S=surface of plate supported by one stay, in square inches.

The surface of plate, S, supported by each stay will evidently be $7\frac{3}{4}\times 7\frac{3}{4}=60{\cdot}1$ square inches.

The thickness of plate, t, in sixteenths of an inch, is 9.

Hence,

$$p=\frac{100(9+1)^2}{60{\cdot}1-6}$$

$$=\frac{100\times 10^2}{54{\cdot}1}=\underline{185 \text{ lbs. per square inch.}}$$

The plate is thus capable of withstanding a safe working pressure of 185 lbs. per square inch.

Chapter VI

RIVETED JOINTS

77.—In determining the internal pressure which can be carried by plain cylindrical vessels, such as steam boilers, we assumed the plates to be solid throughout, i.e. to have no joints.

Fig. 25

As, however, cylindrical shells are generally made from originally flat plates, it is quite obvious that after rolling the plates to the cylindrical form the adjoining edges must be securely jointed together in some manner. The most satisfactory method of doing this is by riveting, which consists of drilling holes near the edges of the plates, lapping one edge over the other, or else butting the two edges together and placing cover strips over, and then passing rivets through. This will be understood from Figs. 25 and 26. In the former figure, one edge of the plate is shown lapped over the other, and in the latter the edges are butted together and are then covered by strips known as "butt straps," the rivets being passed through both the straps and the plate.

Fig. 26

These joints are known as *riveted* joints, that of Fig. 25 being termed a *lap joint,* and the other one a *butt joint.*

Riveted joints may be either single, double, or treble riveted, depending on whether one, two, or three rows of rivets be used for making the joint. The different types will be illustrated as we go along.

We have previously found that the internal pressure which may be safely sustained by a thin cylindrical shell is given by the relation

$$p=\frac{2tf_t}{d},$$

where p=safe or working pressure, in lbs. per square inch,
t=thickness of plates, in inches,
d=diameter of shell, in inches,
f_t=allowable tensile stress in plates, in lbs. per square inch.
This relation supposes that the plates are solid throughout; in other words, it assumes there are no riveted joints.

We must consider now what will be the effect of a riveted joint on the strength of the vessel.

A joint of this type, as we have seen, involves the drilling of holes through the solid plate, and it is quite obvious that after the holes have been drilled the amount of metal to resist the bursting pressure will be less than it was prior to the drilling. The provision of a joint must therefore weaken the structure, with the result that the latter cannot sustain the same pressure as it could if it had no joints.

In order to determine the safe working pressure with the joints taken into account, it is necessary to introduce into the foregoing relation a factor k, which represents the ratio of the actual strength of the joint to that of the solid plate. Thus,

$$p=\frac{2tf_t}{d}\times k,$$

and from this,
$$f_t=\frac{pd}{2tk}.$$

As regards tearing of the plates, the factor k, which, for tearing, may be substituted by k_t, represents the ratio of the amount of metal left after drilling the rivet holes to the amount of metal originally in the solid plate.

If we let D represent the diameter of the rivets, P the pitch of the rivets, i.e. the distance from the centre of one rivet to the centre of the next, and t the thickness of the plates, then considering a strip of plate of length P, the amount of metal remaining to resist tearing after drilling will be $(P-D)t$; the metal in the solid plate originally was $P\times t$.

Hence,
$$k_t=\frac{(P-D)t}{Pt}=\frac{P-D}{P}.$$

This ratio represents the fractional strength of the joint to resist tearing along the line of rivet holes. If it be multiplied by 100, it represents the strength expressed as a percentage, and this percentage strength is commonly known as the *efficiency* of the joint.

Thus, $$\text{Tearing efficiency of joint}=\frac{P-D}{P}\times 100.$$

The tearing efficiency of a riveted joint depends on the type of the joint. It is, for instance, less for a single-riveted than for a double-riveted lap joint, because in the former the proportion of metal removed along a single line of rivet holes in drilling the holes is greater than in the latter joint. Approximate values are 60 per cent. and 70 per cent. respectively for the single- and the double-riveted lap joints met with in actual boiler work. To find then the pressure when k_t and the other factors are known we use the relation,

$$p=\frac{2tf_t}{d}\times k_t.$$

If the pressure be known and the tensile stress in the plates be required, we use the general relation,

$$f_t=\frac{pd}{2tk_t}.$$

78.—**Strength of various forms of Riveted Joints.** When a cylindrical vessel constructed of a continuous ring of plate is

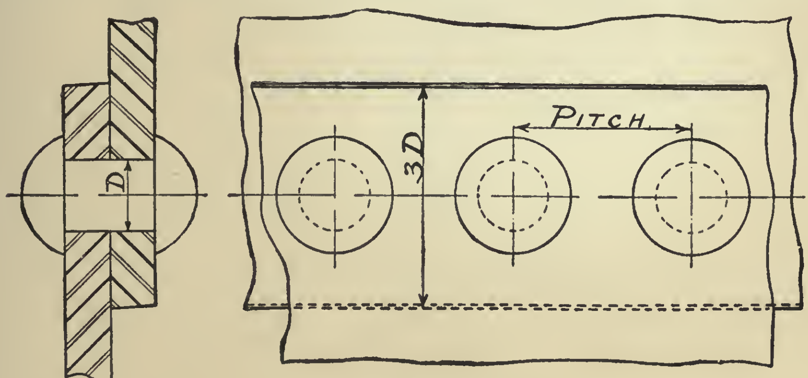

Fig. 27

exposed to internal fluid pressure, it can only fail by tearing through the plate, and if we assume the plate to be of uniform thickness and strength throughout, the vessel would tend to rupture all round its circumference at the same instant.

If, however, the vessel has longitudinal riveted joints, failure is certain to occur at the joint owing to the fact that the latter is always weaker than the solid plate.

Now a riveted joint may fail in quite a number of ways. Thus it may fail :—

(1) By tearing the plate along the line of rivet holes.

(2) By shearing the rivets.

(3) By crushing the rivets, or the plate in front of them. (If of course the rivets be strong enough to resist crushing, then for the same bearing area, the plates, being generally made of the same material, will also be strong enough, and it is sufficient therefore to deal with the rivets only.)

(4) By the plate breaking through between the rivet holes and the edge of the plate in a line at right angles to the edge.

(5) By shearing out the plate in front of the rivets.

To make matters quite clear, consider the simplest type of joint, viz. the *single-riveted lap joint.* Such a joint is illustrated in Fig. 27.

We have seen that the tendency of the pressure acting inside a cylindrical vessel is to burst the vessel longitudinally into two equal halves. If then the single-riveted lap joint of Fig. 27 be the longitudinal joint of such a vessel, it will be understood that there will be two equal and opposite forces acting on the joint in the manner illustrated by Fig. 28, and these two forces tend to cause failure in any one of the five ways referred to.

Fig. 28

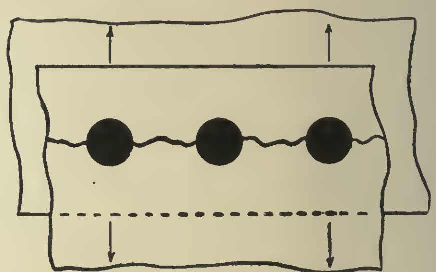

Fig. 29

The various methods by which the joint may fail are illustrated by Figs. 29, 30, 31, 32, and 33.

Fig. 29 shows how the plate may tear along the line of least resistance, i.e. along the line of rivet holes.

Fig. 30 shows failure taking place as a result of the rivets shearing.

Failure by crushing of the rivets is illustrated in Fig. 31.

Fig. 32 illustrates how the plate may break through between the rivet holes and the edge of the plate in a line at right angles to the edge.

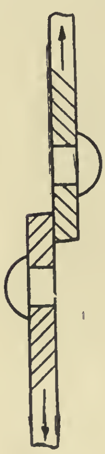

Fig. 30

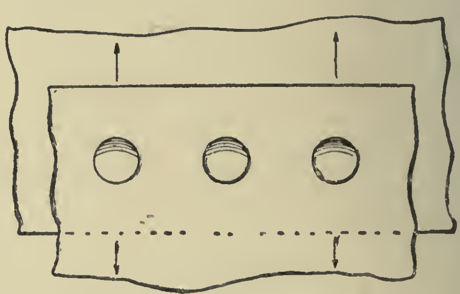

Fig. 31

Finally, Fig. 33 shows how the plate in front of the rivets may be sheared out, thus causing the joint to fail.

79.—The student will understand that in order to have the joint designed to the best advantage it should be so proportioned that it will fail simultaneously in all the five ways just mentioned. If, for instance, the number and diameter of the rivets be such that the joint will fail by tearing of the plate far sooner than it will fail by shearing through the rivets, it is quite obvious that a certain amount of rivet section has been put into the design which from the strength point of view is useless, and which consequently might have been saved. The essence of Machine Design is so to proportion the various parts of a machine or structure that the requisite strength is obtained with the minimum amount of material.

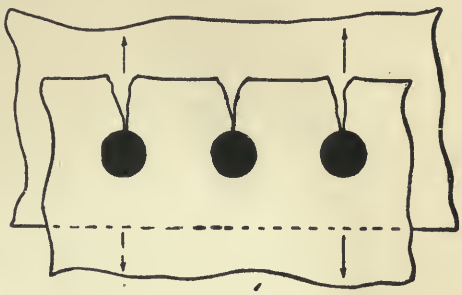

Fig. 32

It has already been explained how the weakening effect caused by the introduction of a riveted joint into a cylindrical vessel subjected to internal pressure is taken into account in calculating the strength of the vessel to resist tearing of the plates, viz. by introducing a factor which represents the ratio of the area of plate left after drilling the holes to the area of the solid plate.

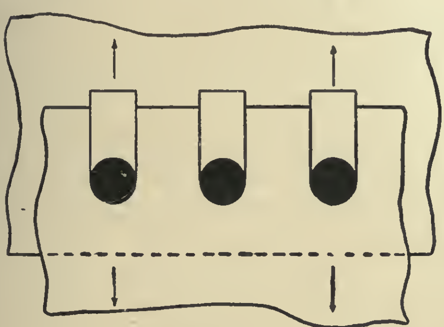

Fig. 33

In the same way, it is convenient to introduce another factor when calculating the strength of the vessel to resist failure by shearing of the rivets. Before failure can occur in this way, a certain number of rivets per pitch, depending on the type of the joint, must be sheared through. The area of metal presented by these rivets to resist shearing may be compared with the area of metal in the solid plate to resist tearing, but it must not be forgotten that the shear strength of iron or steel is not the same as the tensile strength.

Let $$k_s = \frac{\text{area of rivet section to resist shearing}}{\text{area of solid plate to resist tearing}}.$$

Then if, in the general formula, $$f = \frac{pd}{2t},$$

we introduce k_s, thus, $$f = \frac{pd}{2tk_s},$$

f will represent the shearing stress on the rivets.

Hence, calling f_s the shearing stress on the rivets, we have

$$f_s = \frac{pd}{2tk_s}.$$

That this is so may be shown as follows:—

Considering a length of shell equal to one pitch, the total force tending to burst the shell is $pd\text{P}$, p being the pressure, d the diameter of the vessel, and P the pitch of the rivets. If there be n rivets to a pitch, the area to resist shearing (assuming the rivets to be in single shear, and remembering that there are two strips of shell to resist bursting) will be $n\times\frac{\pi}{4}\times D^2\times 2$, where D is the diameter of the rivets. The shearing stress on the rivets will then be

$$f_s=\frac{\text{total force}}{\text{rivet area}}=\frac{pdP}{n\frac{\pi}{4}D^2 2}=\frac{pdP}{n\frac{\pi}{2}D^2}.$$

Now $$k_s=\frac{\text{rivet area to resist shearing}}{\text{area of solid plate to resist tearing}}=\frac{n\times\frac{\pi}{4}\times D^2}{Pt}.$$

Substituting this value of k_s in the equation, $f_s=\frac{pd}{2tk_s}$, we have

$$f_s=\frac{pdPt}{2tn\frac{\pi}{4}D^2}=\frac{pdP}{2n\frac{\pi}{4}D^2}=\frac{pdP}{n\frac{\pi}{2}D^2}.$$

This is just the same result as we obtained by dividing the total force per pitch tending to burst the shell by the total rivet area resisting shearing.

In a similar manner, the crushing stress on the rivets may be found from the general equation for the strength of a cylindrical shell having no joints, by introducing a factor k_c, this factor representing the ratio of the area of rivet metal to resist crushing to the area of solid plate to resist tearing.

Thus, if $$k_c=\frac{\text{area of rivet metal to resist crushing}}{\text{area of solid plate to resist tearing}},$$

the crushing stress $f_c=\frac{pd}{2tk_c}$.

To sum up, then, if we require to find the tensile stress in the plates, the shearing stress on the rivets, or the crushing stress on the rivets of a vessel of cylindrical form exposed to internal fluid pressure, we use the general equation

$$f=\frac{pd}{2tk}.$$

If it be the tensile stress in the plates which is required, we use k_t *for* k, k_t *representing the ratio of plate left between the rivet holes to the solid plate; if the shearing stress on the rivets is to be found, we substitute* k_s *for* k, *where* k_s *represents the ratio of rivet section (to resist shearing) to the solid plate.*

Lastly, if we wish to determine the crushing stress on the rivets, we substitute k_c *for* k, *where* k_c *represents the ratio of rivet metal available to resist crushing to the amount of solid plate to resist tearing.*

80.—With regard to the failure of a riveted joint by the plate breaking through between the rivet holes and the edge of the plate, and by shearing out the plate in front of the rivets, it is customary in modern practice to make the distance between the centre of the rivets and the edge of the plates about one and a half times the diameter of the rivets, as by so doing failure in the two ways referred to is not then likely to occur.

To calculate the strength of the plate to resist breaking through between the rivet holes and the edge, we have to assume that the piece of metal in front of each rivet is a form of short beam fixed at both ends and loaded more or less uniformly along its length. By means of certain relations which are known to hold good in connection with loaded beams, but which the student is not yet sufficiently advanced to deal with, we can then determine what depth of metal there should be between the hole and the plate edge in order that the strength to resist breaking through will be equal, say, to the strength of the plate to resist tearing along the line of rivet holes. We should find this to be considerably less than the diameter of the rivets, so that by making the distance between the centre of the rivets and the edge of the plate one and a half times the rivet diameter, which of course means making the distance between the hole and the plate edge equal to one rivet diameter, we ensure that the joint cannot fail by breaking through in the manner referred to.

As regards failure by shearing out the plate in front of the rivets, before this can occur, two strips of metal of length $1\frac{1}{2}$D and thickness t (where D and t are the diameter of the rivet and the thickness of the plate respectively) would have to be sheared through, and the resistance offered by these strips is generally much greater than that offered by the joint to resist failure in other ways. As a matter of fact, a riveted joint is seldom, if ever, found to fail by the plate in front of the rivets being sheared out.

Generally speaking, then, when designing a riveted joint, it is not necessary to pay attention to the strength of the joint to resist breaking through between the rivet holes and the edge of the plate, or to resist shearing out the plate in front of the rivets, providing the distance between the centre of the rivets and the edge of the plate be made equal to one and a half times the diameter of the rivets. The strengths to be considered are those to resist tearing of the plate through the rivet holes, shearing of the rivets, and crushing of the rivets.

The usual procedure when designing riveted joints for steam boilers, etc., is as follows:—Having determined the thickness of plate required, the diameter of the rivets is fixed. The resistance to tearing through the plate is next calculated, and then the resistance to shearing. By equating these two resistances, the necessary pitch of the rivets can be determined. Finally, the crushing resistance of the rivets is calculated to ensure that with the pitch now decided upon, the strength of the rivets to resist crushing is equal to their strength to resist shearing, or to the strength of the plate to resist tearing. If not, it would be necessary to modify the design somewhat, unless, of course, there is an ample margin of strength. It may be observed that as the plates are gripped very tightly together by the rivets, an additional resistance to failure of the joint is thus presented, but it is not usual to take any account of this in calculating the strength of the joint.

The diameter of the rivets is commonly fixed by means of a simple formula due to Unwin, viz. :—

$$D=1{\cdot}2\sqrt{t},$$

where D=diameter of rivet and t=thickness of plates.

Instead of using a formula, however, many structural engineers adopt a certain size of rivet for a certain thickness of plate, as, for example, a $\frac{3}{4}$-inch rivet for a $\frac{3}{8}$-inch plate, a $\frac{7}{8}$-inch rivet for a $\frac{1}{2}$-inch plate, and so on.

Mr. Edward G. Hiller, Chief Engineer of the National Boiler and General Insurance Company, Limited, Manchester, recommends the following sizes of rivets for various thicknesses of boiler plates.

Thickness of Plate.	Diameter of Rivet Hole. Finished Size of Rivet.
$\frac{3}{8}$ inch.	$\frac{3}{4}$ inch.
$\frac{7}{16}$ "	$\frac{13}{16}$ "
$\frac{1}{2}$ "	$\frac{13}{16}$ "
$\frac{9}{16}$ "	$\frac{13}{16}$ "
$\frac{5}{8}$ "	$\frac{7}{8}$ "
$\frac{11}{16}$ "	$\frac{7}{8}$ "
$\frac{3}{4}$ "	$\frac{15}{16}$ "
$\frac{13}{16}$ "	1 "
$\frac{7}{8}$ "	1 "
$\frac{15}{16}$ "	$1\frac{1}{16}$ "
1 "	$1\frac{1}{16}$ "

81.—**Single-Riveted Lap Joint.** This is the simplest type of riveted joint; it is made by lapping the edge of one plate over the edge of the same (or another) plate, and then fastening the two together by a single row of rivets. (See Fig. 27.)

In considering the strength of any riveted joint, it is only necessary to take a length of plate equal to one pitch.

Referring to the figure, there are forces acting on the two plate edges, as indicated by the arrows in Fig. 28. These forces, which are, of course, due to the internal pressure in the vessel to which the joint belongs, are tending to tear the plate along the line of rivet holes, and also to shear and crush the rivets in the manner already explained.

The amount of plate metal per pitch resisting tearing is $(P-D)t$, and the amount of rivet metal resisting shearing is $\frac{\pi}{4}D^2$, i.e. the area of two half rivets or one whole rivet.

Calling the ultimate tensile stress of the plate metal f_t^1, and the ultimate shearing stress of the rivet metal f_s^1, the tearing resistance is $(P-D)tf_t^1$, and the shearing resistance $\frac{\pi}{4}D^2f_s^1$.

As it is desirable so to design the joint that it will be equally strong to resist both tearing and shearing, we equate the tearing and shearing resistances. Thus

$$(P-D)tf_t^1=\frac{\pi}{4}D^2f_s^1.$$

From this equation, we can find the required pitch, P, because D, t, f_t^1, and f_s^1 are supposed to be known.

Dividing both sides of the equation by tf_t^1, we have

$$P-D=\frac{\pi D^2f_s^1}{4tf_t^1}.$$

Then
$$P=\frac{\pi D^2f_s^1}{4tf_t^1}+D.$$

The pitch is thus obtained by equating the tearing and shearing resistances of the joint. It must be pointed out, however, that the pitch as determined in this way may in some instances require adjustment to suit practical requirements. Thus, if it be too small, so that the rivets come too close together, the operation of riveting may be interfered with; whilst if it be too large the plates cannot be closed properly to obtain a thoroughly tight joint.

The fractional strength of the joint for tearing, k_t, is the proportion of the metal left after drilling the rivet holes to the metal in the solid plate, and taking one pitch, this is

$$\frac{(P-D)t}{Pt}=\frac{P-D}{P}.$$

Hence,
$$k_t=\frac{P-D}{P}.$$

The value of the shearing factor, k_s, which, as already explained, represents the proportion of the metal to be sheared through to the metal in the solid plate is

$$\frac{\frac{\pi}{4}D^2}{Pt}=\frac{\pi D^2}{4Pt}.$$

[Note that there is one rivet (or two halves) resisting shearing per pitch.] Therefore,

$$k_s=\frac{\pi D^2}{4Pt}.$$

As regards crushing of the rivets, there is again one rivet to each pitch, and the ratio of the amount of rivet metal to resist crushing to the amount of metal in the solid plate is

$$\frac{D\times t}{Pt}=\frac{D}{P}.$$

Hence,
$$k_c=\frac{D}{P}.$$

(Note that the area of rivet metal to resist crushing is not the half circumference of the rivet multiplied by the thickness of plate, but the diameter by the thickness.)

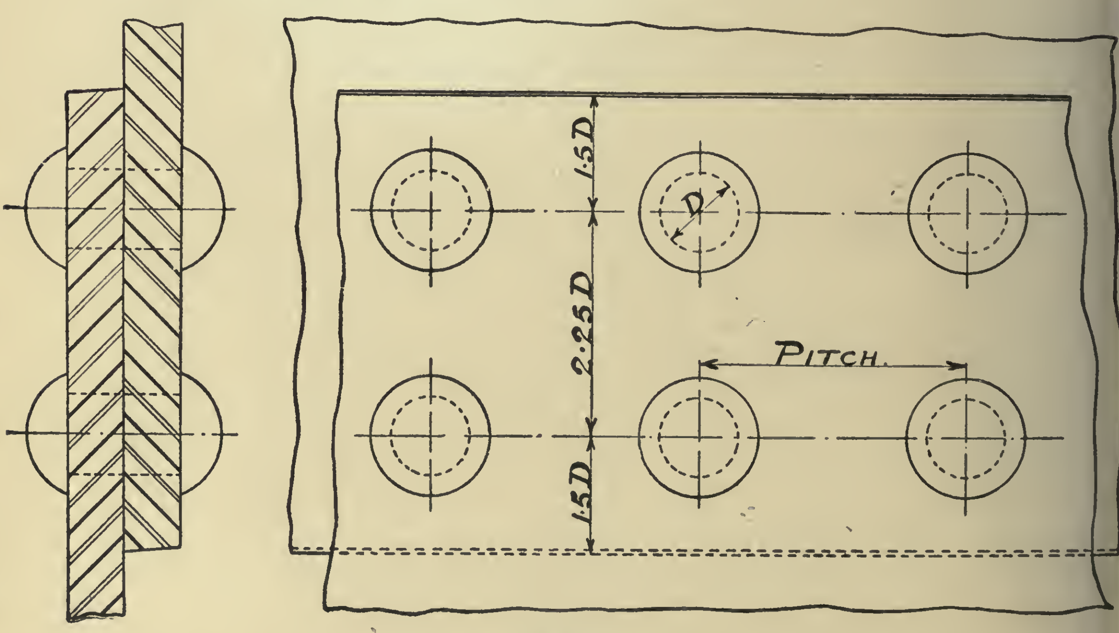

Fig. 34

82.—**Double-Riveted Lap Joint.** A double-riveted lap joint is similar to a single-riveted joint, but two rows of rivets are employed instead of a single row. Fig. 34 shows such a joint, the rivets in this case being arranged in the form known as "chain riveting."

The tearing resistance of the plates in a longitudinal direction for this class of joint is clearly $(P-D)tf_t^1$ per pitch, and the shearing resistance of the rivets $2\times\frac{\pi}{4}D^2f_s^1$, because there are now two rivets to a pitch to be sheared before failure by shearing of the rivets can take place.

It follows at once that if the tearing resistance of the plates is to be equal to the shearing resistance of the rivets, a larger pitch may be adopted for a double- than for a single-riveted lap joint, as there are just twice as many rivets to shear through per pitch in the double as in the single joint. Further, the adoption of a larger pitch means that there will be a less number of rivet holes in each line of rivets, and consequently a smaller amount of metal removed by drilling than is the case when a smaller pitch is adopted. (It is to be understood of course that failure takes place longitudinally through a single line of rivet holes.) The joint is stronger in consequence. This is the reason why a double-riveted joint is stronger than a single-riveted joint; it is not merely because there are more rivets in it, as is commonly supposed.

Equating the tearing resistance of the plates to the shearing resistance of the rivets, we have

$$(P-D)tf_t^1=2\frac{\pi}{4}D^2f_s^1.$$

Dividing through by tf_t^1,

$$(P-D)=\frac{\pi D^2f_s^1}{2tf_t^1}.$$

Hence,
$$P=\frac{\pi D^2f_s^1}{2tf_t^1}+D.$$

The amount of plate per pitch left after drilling the rivet holes is $(P-D)t$, and the amount of the solid plate is Pt. Therefore,

$$k_t=\frac{(P-D)t}{Pt}=\frac{P-D}{P}.$$

The ratio of the amount of rivet section to resist shearing to the solid plate is $2\frac{\pi}{4}D^2$ to Pt. Therefore,

$$k_s=\frac{\pi D^2}{2Pt}.$$

The ratio of rivet metal to resist crushing to the solid plate is $2\times D\times t$ to Pt. Consequently,

$$k_c=\frac{2Dt}{Pt}=\frac{2D}{P}.$$

Fig. 35 shows a double-riveted lap joint with the rivets arranged "zig-zag." For boiler work, the rivets are usually arranged in this way in preference to the chain form of riveting, as the zig-zag riveting makes a tighter joint, and also does not require so wide a lap as the chain form.

Referring to the figure, it will be observed that the amount of metal per pitch resisting tearing between the rivet holes in a longitudinal direction is the same as that for the joint of Fig. 34, viz. $(P-D)t$. There are also two rivets to be sheared through (or two halves and a whole one per pitch), and the same number to

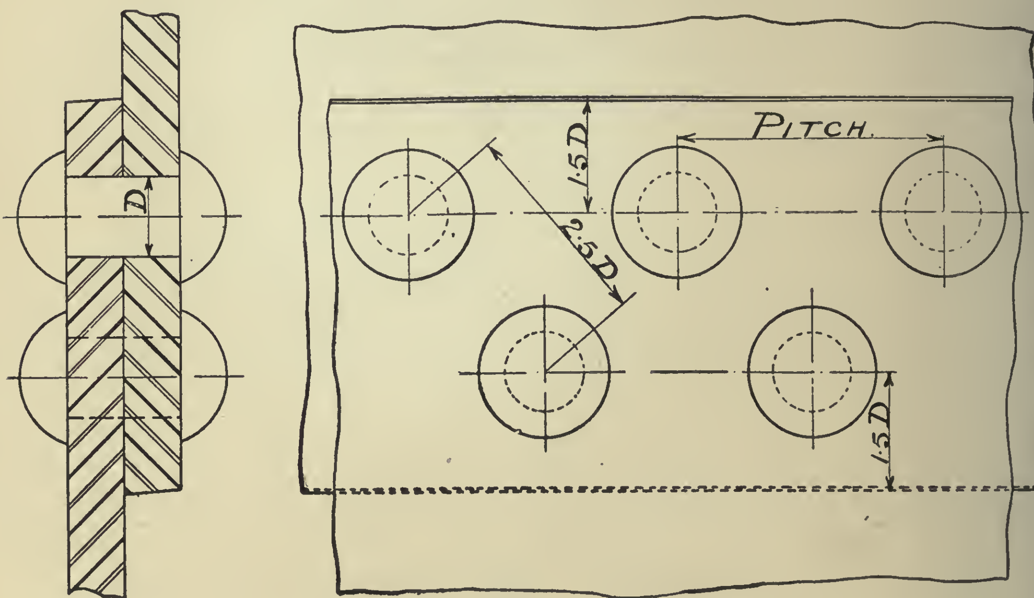

Fig. 35

be crushed before failure by shearing or crushing can occur. The pitch, the values of k_t, k_s, and k_c will therefore all be precisely the same as for the chain double-riveted lap joint, viz.

$$P=\frac{\pi D^2 f_s^1}{2tf_t^1}+D,$$

$$k_t=\frac{P-D}{P},$$

$$k_s=\frac{\pi D^2}{2Pt},$$

$$k_c=\frac{2D}{P}.$$

In addition to the longitudinal pitch of the joint under consideration, we have also to determine the diagonal pitch of the rivets, i.e. the distance from the centre of any one rivet to the centre of the next, measured diagonally. If this diagonal pitch be made too small, the joint may fail by the plates tearing across in zig-zag fashion. Assuming the strength of the plate to be the same in all directions, the same for instance with the grain as across the grain, then the amount of metal between the holes, measured diagonally, might be made equal to one half the amount between the holes, measured longitudinally. The plate is stronger to resist tearing across the grain than with the grain, and as the latter runs in a direction at right angles to the longitudinal seams, that is to say, in the direction in which the plates are rolled originally, the plate is stronger in a longitudinal direction than in a diagonal direction. More metal should therefore be allowed for in a diagonal than in a longitudinal direction, and, consequently, the shortest distance between two holes, measured diagonally, must be more than one half the shortest distance between two holes measured longitudinally. Equal strength in the two directions is obtained by making the former distance two-thirds the latter.

Various rules are used for fixing the diagonal pitch. According to Professor Kennedy, who made important experiments with riveted joints, the net section of metal in the plate, measured diagonally, should be from 30 to 35 per cent. in excess of that measured longitudinally; this gives a diagonal pitch of $\frac{2}{3}P+\frac{1}{3}D$.

For boiler work, Mr. Edward G. Hiller states that the diagonal pitch should not be less than $\cdot65P+\cdot35D$. In actual boiler work, the diagonal pitch is commonly found to be approximately $2\cdot5D$, as marked on the sketch.

The Board of Trade fix the shortest distance, V, between the two rows of rivets, by the rule,

$$V=\frac{\sqrt{(11P+4D)(P+4D)}}{10}.$$

This of course fixes the diagonal pitch.

83.—**Treble-Riveted Lap Joints.** A lap joint is said to be treble-riveted when it is composed of three rows of rivets. The rivets may be arranged either as chain or zig-zag riveting, as in the case of the double-riveted lap joint.

Treble-riveted lap joints are not much used in actual practice, but, nevertheless, it may be advisable to consider briefly the strength of these joints.

Whether the riveting be arranged in the chain or the zig-zag form, the amount of metal per pitch (for any one row of rivets) available to resist tearing between the rivet holes will be $(P-D)t$, whilst the number of rivets per pitch to resist shearing and crushing will be three. The tearing resistance is therefore $(P-D)tf_t^1$ and the shearing resistance $3\frac{\pi}{4}D^2f_s^1$.

Equating the two resistances,

$$(P-D)tf_t^1 = \tfrac{3}{4}\pi D^2 f_s^1.$$

Dividing through by tf_t^1,

$$P-D = \frac{3\pi D^2 f_s^1}{4tf_t^1}.$$

$$\therefore\ P = \frac{3\pi D^2 f_s^1}{4tf_t^1} + D.$$

The diagonal pitch for the zig-zag treble-riveted joint is determined in exactly the same way as it is for the double-riveted joint.

The values for the constant k are as follows:—

$$k_t = \frac{P-D}{P},$$

$$k_s = \frac{3\pi D^2}{4Pt},$$

$$k_c = \frac{3D}{P}.$$

84.—**Single-Riveted Butt Joint.** A single-riveted butt joint is illustrated in Fig. 36. The left-hand section shows the joint with a single cover strap only, and the right-hand section with double cover straps. There is a single row of rivets on each side of the plate joint.

As regards the single strap joint, this is really equivalent to two distinct single-riveted lap joints, and the pitch, the values of k_t, k_s, and k_c are determined in the manner explained in connection with single-riveted lap joints. When the joint belongs to a cylindrical vessel under pressure, the pull on the plates tends to bend the strap, and to make allowance for the stress set up by the bending action the thickness of the strap should be made greater than that of the plates, say one and an eighth to one and a quarter times as great.

In the case of the double strap joint, it is to be noted that each rivet, instead of being in single shear, as in all the joints so far considered, is in double shear. That is to say, before the joint can fail by shearing of the rivets, each rivet will have to be sheared across two sections. The result is that so far as failure by shearing is concerned, the joint is approximately twice as strong as it would be if the rivets were in single shear only.

Some authorities consider that a rivet in double shear does not offer twice the resistance it would do if only in single shear; the Board of Trade, for example, assume the strength of a rivet in double shear to be one and seven-eighths times as great as when in single shear. Experiments carried out by Professor Kennedy, however, have shown that the relative strengths are approximately two to one, and in what follows, therefore, except where otherwise stated, we shall take it that a rivet in double

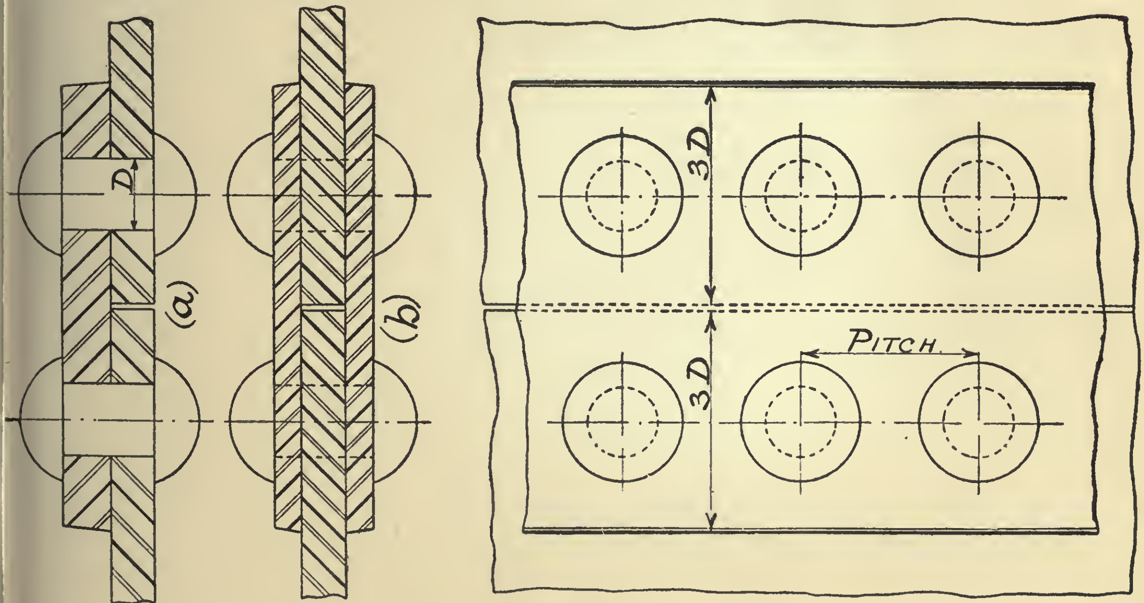

Fig. 36

shear is just twice as strong to resist failure by shearing as it is when in single shear.

Referring now to the single-riveted butt joint with double straps, we see that the tearing resistance of the plates is $(P-D)tf_t^1$.

The shearing resistance of the rivets is $\frac{\pi}{4}\times D^2\times f_s^1\times 2$, i.e. one rivet in double shear per pitch.

Equating the two resistances, we have

$$(P-D)tf_t^1=\frac{\pi}{4}D^2f_s^1 2.$$

Dividing through by tf_t^1, we get

$$P-D=\frac{\pi}{2}\frac{D^2f_s^1}{tf_t^1},$$

from which,
$$P=\frac{\pi}{2}\frac{D^2f_s^1}{tf_t^1}+D.$$

The values of the constants are easily seen to be

$$k_t = \frac{P-D}{P},$$

$$k_s = \frac{\pi D^2}{2Pt},$$

$$k_c = \frac{D}{P}.$$

As regards the thickness of the straps, it is customary to make each strap rather thicker than one half the thickness of the plates. If, of course, the combined thickness be less than that of the plates, the value of k_c will be reduced.

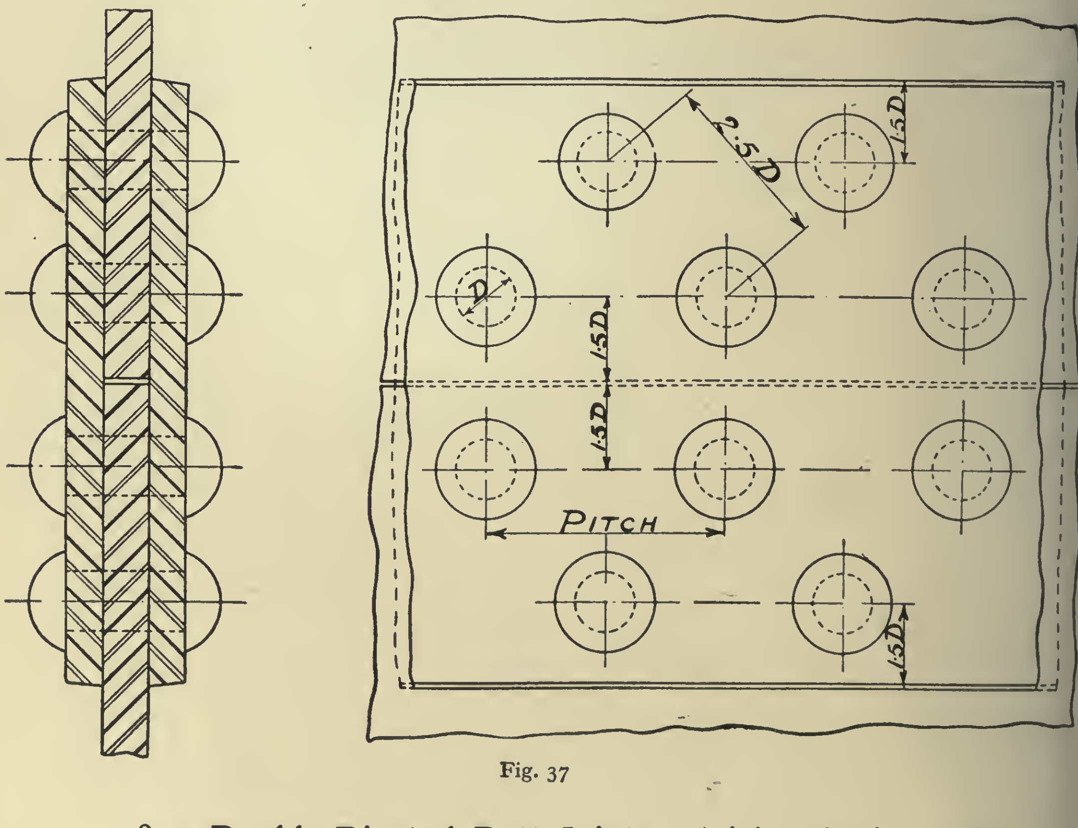

Fig. 37

85.—**Double-Riveted Butt Joints.** A joint of this type is shown in Fig. 37. Double-riveted butt joints are generally made with double cover straps, and the riveting is mostly of the zig-zag form. We shall only consider therefore this form of double-riveted butt joint. The diagonal pitch of the riveting, it should be mentioned, is determined by the rules already given in connection with double zig-zag riveted lap joints.

Referring to Fig. 37, it is seen that the joint may fail by tearing through the plate along either of the outer rows of rivet holes, or by shearing through two rows of rivets on either side of the plate joint. As the thickness of the plates and the diameter of the rivets are the same on both sides of the plate joint, it is only necessary to consider one side of the joint.

As regards failure by tearing, the amount of plate per pitch is $(P-D)t$, so that the tearing resistance is $(P-D)tf_t^1$.

Before failure by shearing can occur, two rivets per pitch in double shear will have to be sheared. The shearing resistance is therefore,

$$2\frac{\pi}{4}D^2f_s^1 2=\pi D^2f_s^1.$$

For equal strength the tearing resistance must be equal to the shearing resistance, and hence

$$(P-D)tf_t^1=\pi D^2f_s^1$$

$$\therefore\ P=\frac{\pi D^2f_s^1}{tf_t^1}+D.$$

The ratio of the amount of plate between the rivets to the amount of the solid plate is $\frac{P-D}{P}$.

The ratio of rivet section to the solid plate is

$$\frac{2\frac{\pi}{4}D^2 2}{Pt}=\frac{\pi D^2}{Pt}.$$

The ratio for crushing is

$$\frac{2Dt}{Pt}=\frac{2D}{P}.$$

Hence,
$$k_t=\frac{P-D}{P};$$

$$k_s=\frac{\pi D^2}{Pt};$$

$$k_c=\frac{2D}{P}.$$

As regards the butt straps, the thickness of each of these may be made equal to from $\frac{5}{8}t$ to t.

86.—**Treble-Riveted Butt Joints.** For modern high pressure boiler work, treble-riveted butt joints, with double cover straps, are nearly always used for the longitudinal seams. Two slightly different forms of this type of joint are employed, the difference being simply in the width of the outer cover strap. The two forms are shown in Fig. 38. The section (*a*) shows the upper or outer cover strap to be narrower than the lower one, the former being only wide enough to take two rows of rivets on each side of the joint, whilst the latter takes three rows. The section (*b*) shows both straps of the same width, and wide enough to take three rows of rivets on each side of the joint. The plan has been drawn for the section (*a*), but with very slight modification it will also represent the plan of section (*b*).

In both forms of this joint it will be observed that the pitch of the outer rows of rivets is made twice the pitch of the inner rows. By leaving out alternate rivets in the outer rows in this way, a stronger joint is obtained, less metal being thus removed by drilling from the part where failure would occur by tearing than would otherwise be the case. The real pitch of the joint is the large one.

We shall consider first the joint represented by the section (*b*).

This joint may fail by tearing through the plate along an outer row of rivet holes, by tearing through the plate along a middle row, and at the same time shearing the outer rivets, or by shearing all the rivets, etc. For the proportions generally adopted in practice, failure in the second way is not liable to occur. We need only consider, therefore, failure by tearing along one of the outer rows of rivet holes, and by shearing of all the rivets.

The tearing resistance is $(P-D)tf_t^1$. To resist failure by shearing of the rivets, there are five rivets in double shear per pitch available. The shearing resistance is then $5\frac{\pi}{4}D^2f_s^1 2$.

Equating the two resistances, we have

$$(P-D)tf_t^1=5\frac{\pi}{4}D^2f_s^1 2=\frac{5}{2}\pi D^2f_s^1,$$

$$(P-D)=\frac{5\pi D^2f_s^1}{2tf_t^1},$$

from which,

$$P=\frac{5\pi D^2f_s^1}{2tf_t^1}+D.$$

The values of the constants are

$$k_t=\frac{P-D}{P},$$

$$k_s=\frac{5\pi D^2 2}{4Pt}=\frac{5\pi D^2}{2Pt}.$$

$$k_c=\frac{5Dt}{Pt}=\frac{5D}{P}.$$

The joint of Fig. 38 (*a*) may fail in the same three ways as those mentioned in connection with the joint of Fig. 38 (*b*), viz. by tearing

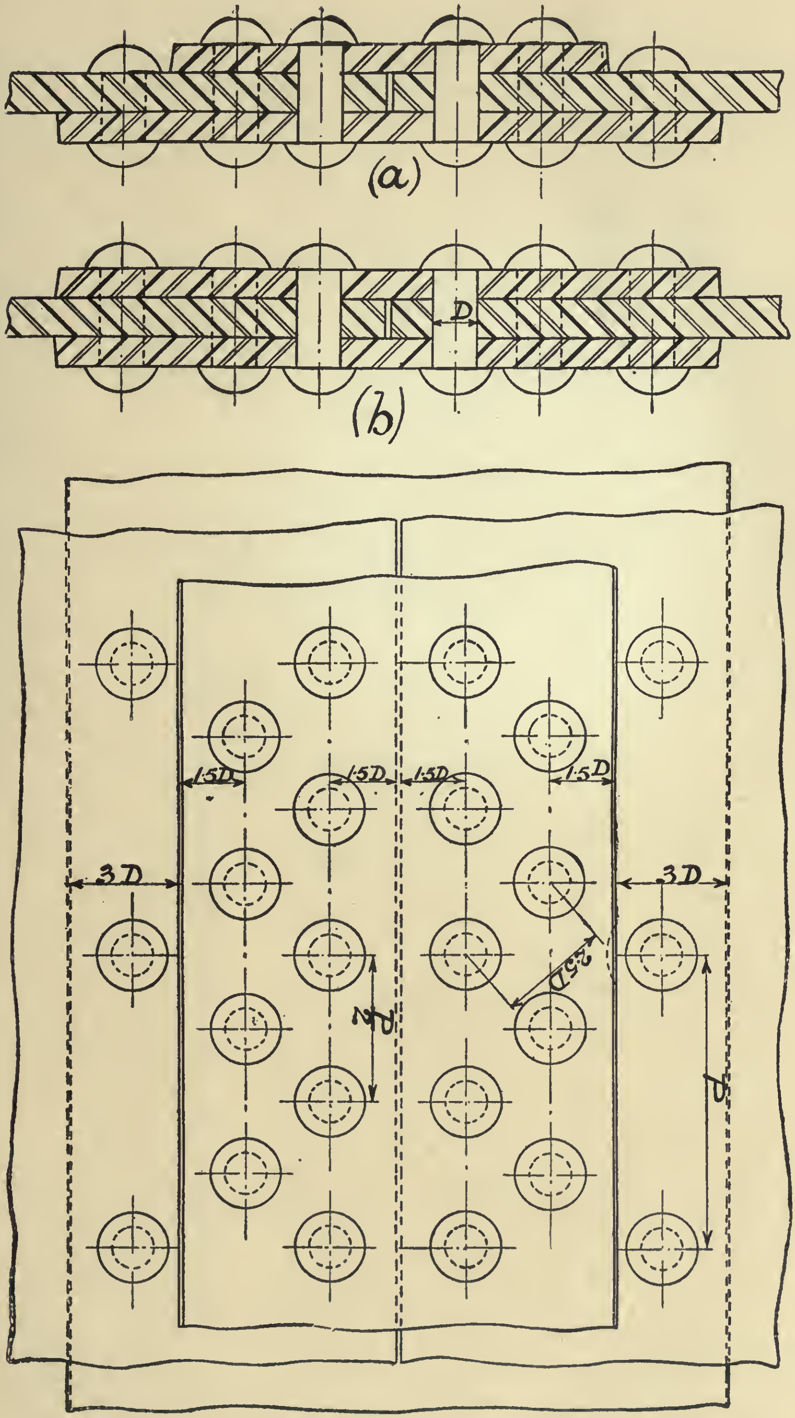

Fig. 38

through the plate along an outer row of rivet holes, by tearing along a middle row and shearing the outer rivets, or by shearing all the rivets. It is to be noted in this case, however, that the outer rivets are in single shear only, and it follows that the assistance furnished by the outer rivets to prevent failure of the joint by tearing along either of the second rows of rivet holes will be less than in the previous case, where the outer rivets are in double shear. Consequently, in a joint of this type, although we generally determine the pitch by equating the tearing resistance along one of the outer rows of rivet holes, to the shearing resistance of the rivets, it is perhaps advisable to check the design to see that the joint is not likely to fail by tearing along either of the inner rows of rivet holes and shearing the outer rivets, rather than by tearing along the outer rows of rivet holes or shearing all the rivets.

The tearing resistance of the plate along the outer rows of rivet holes is $(P-D)tf_t^1$. To resist shearing, there are four rivets in double shear and one in single shear. The shearing resistance is then $4\frac{\pi}{4}D^2f_s^1 2+\frac{\pi}{4}D^2f_s^1=9\frac{\pi}{4}D^2f_s^1$.

Equating,

$$(P-D)tf_t^1=9\frac{\pi}{4}D^2f_s^1$$

$$(P-D)=\frac{9\pi D^2f_s^1}{4tf_t^1},$$

from which, $$P=\frac{9\pi D^2f_s^1}{4tf_t^1}+D.$$

The values of the constants are as follow:—

$$k_t=\frac{P-D}{P},$$

$$k_s=\frac{9\frac{\pi}{4}D^2}{Pt}=\frac{9\pi D^2}{4Pt},$$

$$k_c=\frac{5Dt}{Pt}=\frac{5D}{P}.$$

In a treble-riveted butt joint of the form shown in Fig. 38, where the pitch of the outer rows of rivets is twice the pitch of the inner rows, it is advisable to consider the strength of the cover straps. The forces tending to separate the two edges of the shell plate at the joint also act, through the medium of the rivets, on the straps, which are in consequence put in tension.

The tearing resistance of the two straps at either of the lines of rivet holes nearest the plate joint is $(P-2D)t_sf_t^1\times 2$, where t_s is the thickness of each strap. The strength of the plates to resist tearing is $(P-D)tf_t^1$. (We assume the straps and the plates to be made of the same material.)

Equating these, we have

$$2(P-2D)t_s f_t^1 = (P-D)tf_t^1.$$

Cancelling out f_t^1,

$$2(P-2D)t_s = (P-D)t.$$

$$\therefore\ t_s = \frac{(P-D)t}{2(P-2D)}.$$

This expression gives us the thickness of each strap from theoretical considerations. It has already been explained, however, that allowance should be made for the fact that the load may not be thrown equally upon the two straps, and the actual thickness should consequently be rather more than that given by the above expression.

In the construction of large steam boilers, it is customary to make the thickness of the straps of treble-riveted butt joints almost equal to the thickness of the shell plates; usually about $\frac{1}{16}$ inch or $\frac{1}{8}$ inch less.

So far, no reference has been made to the circumferential joints of cylindrical shells. Except in the case of short vessels, the shell must be constructed of two or more rings, and these are usually connected together by riveted joints.

For the longitudinal seams, the butt joint is preferable to the lap joint, because it retains the true circular form of the shell, and thus obviates certain defects which are liable to arise when lap joints are employed, owing to the fact that the latter interfere with the circularity somewhat.

As regards the circumferential seams, the lap joint is entirely satisfactory and is invariably used in boiler work, the single-riveted joint being employed for low pressures, and the double-riveted joint for high pressures. The calculations for these joints are very simple, and should present no difficulty whatever to the student who has read through the previous work. It is to be remembered, of course, that whereas the total force tending to burst the vessel in a longitudinal direction is pdl, where p represents the pressure, d the diameter of the vessel, and l the length, the total force tending to burst the vessel circumferentially is $p\frac{\pi}{4}d^2$.

If the vessel contains internal tubes, the area on which pressure acts will be less than $\frac{\pi}{4}d^2$ by the sectional area of the tubes. In Lancashire and similar boilers, the internal flue tubes tend to prevent bursting of the boiler in a circumferential direction, but it is not customary to take this into account when designing the joints.

The various types of riveted joint dealt with in the foregoing represent those mostly used in connection with steam boiler work and cylindrical vessels generally, but in addition to these, other joints, although generally similar, but differing somewhat in the arrangement of the rivets, are sometimes met with. The student, however, after studying the joints given, should have no difficulty in dealing with any other joint of a similar nature which may come before him.

87.—**Riveted Joints for Girder Work.** In the design of built-up girders, bridges, etc., a knowledge of the strength of riveted work is as essential as it is in the case of boiler work. Generally speaking, the student who understands the first principles of the design of riveted joints can apply his knowledge equally to boiler work or bridge work. One or two examples of riveted joints in bridge and girder work may, however, be studied with advantage.

Fig. 39 shows what is known as a lap-riveted *tie bar joint.* Here, two flat bars are connected together by lapping the end of one over the end of the other and passing nine rivets through the two ends, which are shaped as shown.

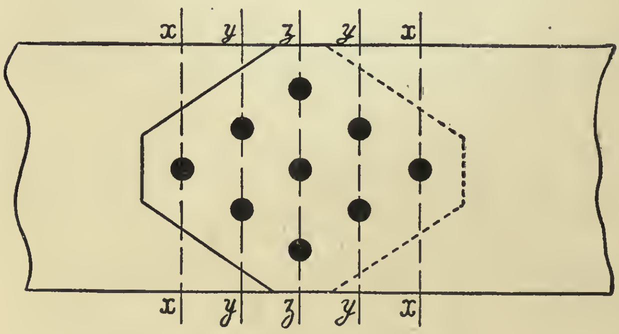

Fig. 39

In designing a joint of this form, the main object is to determine the number of rivets which must be used in order that the joint will be as strong to resist failure by shearing of all the rivets as it is to resist failure by tearing through either bar.

Let b represent the breadth of either bar,
,, t ,, ,, thickness ,, ,,
,, D ,, ,, diameter of the rivets,
,, f_t^1 ,, ,, tensile strength of the plates,
,, f_s^1 ,, ,, shearing strength of the rivets,
,, n ,, ,, number of rivets required.

Then the strength of the bars to resist tearing through either of the sections xx is $(b-D)tf_t^1$, and the strength to resist shearing of all the rivets is $n\frac{\pi}{4}D^2f_s^1$, the rivets being in single shear.

Equating these two strengths, we have

$$(b-D)tf_t^1 = n\frac{\pi}{4}D^2f_s^1.$$

The number of rivets required is then,

$$n = \frac{(b-D)tf_t^1}{\frac{\pi}{4}D^2f_s^1} = \frac{4(b-D)tf_t^1}{\pi D^2 f_s^1}.$$

The joint under consideration might fail by tearing at either of the sections *yy* and shearing the rivet at one of the sections *xx*; by tearing across *zz* and shearing the rivets at one of the sections *yy* and at one of the sections *xx*, and also by crushing of the rivets. After having determined, therefore, the required number of rivets, and spacing them conveniently, it is advisable to calculate the strength of the joint to resist failure in the three ways referred to, and, if necessary, modify the design.

The strength of the joint to resist failure by tearing at either of the sections *yy* and shearing the rivet at one of the sections *xx* is $(b-2D)tf_t^1 + \frac{\pi}{4}D^2f_s^1$.

The strength of the joint to resist failure by tearing across *zz* and shearing the rivets at one of the sections *yy*, and also at one of the sections *xx*, is

$$(b-3D)tf_t^1 + 3\frac{\pi}{4}D^2f_s^1.$$

The strength to resist crushing of the rivets is $9Dtf_c^1$, where f_c^1 represents the crushing strength of the rivets.

The above joint, instead of being made in the lap form, may be made butt-jointed with double cover straps, the ends of the bars being butted against each other, and the straps instead of the bars being shaped at each end to suit the arrangement of rivets. This form of tie bar joint is superior to the lap form, because the pulling forces acting along the tie on opposite sides of the joint act in one and the same plane, whereas in the lap joint, they act in different planes, with the result that a bending action is set up.

The butt joint is also stronger than the lap (as regards its resistance to shearing of the rivets), other things being the same, owing to the fact that the rivets are in double shear, so that for the same pull along the tie bar, a less number of rivets are required than is the case when a lap joint is employed, where the rivets are only in single shear.

The calculations for the strength of the butt joint are similar to those already given for the lap joint, with the exception that the area of rivet section to resist shearing is twice what it is for the corresponding lap joint.

The rivets used in bridge and girder work are usually of a rather less size than those used for boiler work.

For such work, the rule

$$D=1{\cdot}1\sqrt{t}$$

is sometimes made use of, where t=the thickness of the plates.

WORKED EXAMPLES

88.—(1) Determine the thickness of the plates required for a steel boiler shell, 5 feet 6 inches diameter, which is to work at a pressure of 50 lbs. per square inch (1) assuming there are no joints, and (2) assuming there is a longitudinal seam or joint which is single lap-riveted, and which has a tearing efficiency of 60 per cent.

The first part of this problem is easily solved by using the formula

$$f_t=\frac{pd}{2t},$$

where f_t=safe tensile stress, in lbs. per square inch,
p=safe working pressure, in lbs. per square inch,
d=diameter of shell, in inches,
t=thickness of plates, in inches.

Rearranging,

$$t=\frac{pd}{2f_t}$$

Take f_t as 10,000 lbs. per square inch, and substitute the known values.

Then $$t=\frac{50\times5{\cdot}5\times12}{2\times10{,}000}=\underline{{\cdot}165 \text{ inch.}}$$

Required thickness of plate if no joint=$\underline{{\cdot}165 \text{ inch.}}$

This thickness is sufficient for actual strength; but, in practice, plates under $\frac{1}{4}$ inch thick are rarely used.

For the second part of the question, we use the above formula, but we must modify it so as to allow for the weakening which results from the introduction of a riveted joint.

The formula is now,

$$f_t=\frac{pd}{2tk_t}$$

where $$k_t=\frac{\text{actual area of plate to resist tearing}}{\text{area of solid plate}}.$$

The efficiency of the joint is given as 60 per cent., so that $k_t={\cdot}6$.

Hence,
$$t=\frac{pd}{2f_t k_t}.$$
Substituting the values,
$$t=\frac{50\times 5\cdot 5\times 12}{2\times 10{,}000\times \cdot 6}=\underline{\cdot 275\text{ inch.}}$$
Required thickness of plates with single-riveted lap joint $=\underline{\cdot 275\text{ inch.}}$

(2) A steel cylindrical air receiver, constructed of $\frac{3}{8}$-inch plates, is 60 inches diameter, and carries a pressure of 100 lbs. per square inch. The longitudinal seam is double lap-riveted, the pitch of the rivets being $2\frac{1}{4}$ inches and their diameter $\frac{3}{4}$ inch. Show whether or not the vessel is quite safe to work at the pressure imposed upon it.

In order to show whether or not the vessel is safe for its work, we must calculate (1) the tensile stress on the plates; (2) the shearing stress on the rivets; and (3) the crushing stress on the rivets. We assume, of course, that the distance from the centre of the rivets to the edge of the plate is made equal to one and a half times the diameter of the rivets, so that it is not necessary to go into the question of strength as regards breaking through the plate between the rivet holes and the plate edge, and shearing out the plate in front of the rivets.

The tensile stress in the solid plate is given by the relation,
$$f_t=\frac{pd}{2t},$$
where f_t=tensile stress due to the pressure, in lbs. per square inch,
p=pressure, in lbs. per square inch,
d=diameter of receiver, in inches,
t=thickness of plates, in inches.

We require to find, however, the stress in the plates at those parts weakened by the rivet holes, and for this we use the relation,
$$f_t=\frac{pd}{2tk_t},$$
where $$k_t=\frac{\text{actual area of plate to resist tearing}}{\text{area of solid plate}}.$$

Now $k_t=\frac{P-D}{P}$, where P=pitch of rivets in inches and D= diameter of rivets in inches. Then
$$k_t=\frac{2\frac{1}{4}-\frac{3}{4}}{2\frac{1}{4}}=\frac{1\frac{1}{2}}{2\frac{1}{4}}=\cdot 667.$$

Substituting this value of k_t along with the known values of p, d, and t, we get
$$f_t=\frac{100\times 60}{2\times\frac{3}{8}\times\cdot 667}=\underline{12{,}000\text{ lbs. per square inch.}}$$

The tensile stress in the plates is therefore 12,000 lbs. per square inch. An air receiver is not subjected to the same conditions of working as a steam boiler; i.e. there are not the same variations of temperature, or the same tendency to wasting of the plates, etc., and a stress of as much as 14,000 lbs. per square inch may be allowed in such vessels. The air receiver under consideration is consequently quite safe as regards its strength to resist tearing through the plates along a line of rivet holes.

To find the shearing stress on the rivets, we have to find the total amount of metal which has to be sheared before the joint can fail by shearing the rivets. Taking a length of shell equal to one pitch, there will be one whole rivet and two halves, i.e. two full rivets to be sheared.

Calling f_s the shearing stress on the rivets, and letting k_s be the ratio of the amount of rivet section to the area of the solid plate,

$$f_s = \frac{pd}{2tk_s}.$$

Now,

$$k_s = \frac{2 \times \frac{\pi}{4} \times D^2}{Pt} = \frac{\pi \times \cdot 75^2}{2 \times 2 \cdot 25 \times \cdot 375} = 1 \cdot 05.$$

Substituting this and the other known values in the general equation, we get

$$f_s = \frac{100 \times 60}{2 \times \cdot 375 \times 1 \cdot 05} = \underline{7630 \text{ lbs. per square inch.}}$$

The shear stress on the rivets is then 7630 lbs. per square inch.

There is no reason why a shear stress of as much as 11,000 lbs. per square inch should not be allowed in the rivets of an air receiver, so that the joint is amply safe so far as regards its resistance to failure by shearing of the rivets.

The crushing stress on the rivets is given by the relation,

$$f_c = \frac{pd}{2tk_c},$$

where f_c = crushing stress due to the pressure, in lbs. per square inch,

$$k_c = \frac{\text{rivet area to resist crushing}}{\text{area of the solid plate}}.$$

The metal available to resist crushing is equal to the number of rivets per pitch multiplied by the diameter of the rivets and by the thickness of the plates.

For the case in point this is $2 \times \frac{3}{4} \times \frac{3}{8}$. The area of the solid plate per pitch is equal to the pitch by the plate thickness, and this is $2\frac{1}{4} \times \frac{3}{8}$.

Hence,

$$k_c = \frac{2 \times \frac{3}{4} \times \frac{3}{8}}{2\frac{1}{4} \times \frac{3}{8}} = \underline{\cdot 667.}$$

Substituting this and the other known values in the general equation,

$$f_c=\frac{100\times 60}{2\times\frac{3}{8}\times\cdot 667}=\underline{12{,}000\text{ lbs. per square inch.}}$$

Hence, crushing stress in rivets=$\underline{\text{12,000 lbs. per square inch.}}$

The allowable crushing stress may be taken to be at least 20,000 lbs. per square inch, so the joint is also amply strong to resist failure by crushing of the rivets.

Alternative Method.—In case there should be any obscurity with regard to the above method of working the problem, we give below an alternative method.

Consider a length of shell equal to one pitch. The total force tending to burst the shell will then be pDP=100×60×2·25=13,500 lbs.

There are two strips of metal resisting this force, and if there were no holes drilled through these strips, the area of metal available would be

$$\text{P}\times t\times 2=2\cdot 25\times\cdot 375\times 2=1\cdot 687\text{ square inches.}$$

The actual metal available after drilling is

$$(\text{P}-\text{D})t\times 2=(2\cdot 25-\cdot 75)\cdot 375\times 2=1\cdot 125\text{ square inches.}$$

$$\therefore\ \text{Tensile stress in plates}=\frac{13{,}500}{1\cdot 125}=\underline{12{,}000\text{ lbs. per square inch.}}$$

The amount of rivet section to resist shearing is

$$2\times\frac{\pi}{4}\times\text{D}^2\times 2=\pi\text{D}^2=\pi\times\tfrac{3}{4}\times\tfrac{3}{4}=1\cdot 768\text{ square inches.}$$

(Note that we must consider the number of rivets in two strips of metal, as there are two strips resisting failure.)

$$\therefore\ \text{Shear stress on rivets}=\frac{13{,}500}{1\cdot 768}=\underline{7630.}$$

The amount of rivet metal to resist crushing is

$$2\times\text{D}\times t\times 2=2\times\cdot 75\times\cdot 375\times 2=1\cdot 125\text{ square inches.}$$

$$\therefore\ \text{Crushing stress on rivets}=\frac{13{,}500}{1\cdot 125}=\underline{12{,}000\text{ lbs. per square inch.}}$$

89.—(3) Design a single-riveted lap joint for steel plates $\frac{7}{16}$ inch thick.

To design the joint completely it is necessary to determine (1) the diameter of the rivets; (2) the pitch of the rivets; (3) the distance from the centre of the rivets to the edge of the plate.

To determine (1), we may use the relation,

$\text{D}=1\cdot 2\sqrt{t}$, where D=diameter of rivet,
t=thickness of plate.

$$\text{D}=1\cdot 2\sqrt{\cdot 4375}=1\cdot 2\times\cdot 661=\underline{\cdot 792\text{ inch}}=\tfrac{13}{16}\text{ inch nearly.}$$

Diameter of rivets=$\underline{\tfrac{13}{16}\text{ inch.}}$

To determine (2), the pitch of the rivets, we must have the strength of the joint to resist tearing of the plates through the line of rivet holes, equal to its strength to resist shearing of the rivets. Considering a length equal to one pitch, and letting f_t^1 and f_s^1 represent the ultimate tensile strength of the plates and the ultimate shearing strength of the rivets respectively,

Strength to resist tearing $=(P-D)tf_t^1=(P-\frac{13}{16})\frac{7}{16}\times f_t^1$.

Strength to resist shearing $=\frac{\pi}{4}D^2f_s^1=\frac{\pi}{4}\times\frac{13}{16}\times\frac{13}{16}\times f_s^1$,
(one rivet only)

where P = pitch of rivets in inches,

Equating the tearing resistance to the shearing resistance,

$$(P-\tfrac{13}{16})\tfrac{7}{16}\times f_t^1=\frac{\pi}{4}\times\tfrac{13}{16}\times\tfrac{13}{16}\times f_s^1,$$

$$P-\tfrac{13}{16}=\frac{\frac{\pi}{4}\times\frac{13}{16}\times\frac{13}{16}\times f_s^1}{\frac{7}{16}\times f_t^1}.$$

The ultimate shearing strength of the mild steel used in boiler work is approximately ·8 of its ultimate tensile strength, so taking $\frac{f_s}{f_t}$ as being equal to ·8, we get finally,

$$P=\frac{\frac{\pi}{4}\times\frac{13}{16}\times\frac{13}{16}\times\cdot 8}{\frac{7}{16}}+\tfrac{13}{16}$$

$$=\cdot 9475+\cdot 8125=\underline{1\cdot 76\text{ inches.}}$$

Required pitch of rivets = <u>1·76 inches</u>, or, say, <u>$1\frac{3}{4}$ inches.</u>

Having fixed the pitch so that the joint is equally strong to resist failure by tearing of the plate between the rivet holes, or shearing of the rivets, it is necessary to ascertain, if, with this pitch, the joint is quite strong enough to resist failure by crushing of the rivets. There is only one rivet to a pitch to resist crushing. The crushing resistance is therefore $D\times t\times f_c^1$, where f_c^1 represents the ultimate crushing strength of the rivets. If this be more than either the tearing or the shearing resistance, we know that the joint is also safe as regards its strength to resist crushing.

Roughly, the crushing strength of steel rivets is twice the shearing strength.

The ratio of the crushing resistance to the shearing resistance in this case is $D\times t\times f_c^1$ to $\frac{\pi}{4}D^2f_s^1$. Then,

$$\text{Ratio}=\frac{D\times t\times f_c^1}{\frac{\pi}{4}\times D^2\times f_s^1}=\frac{\frac{13}{16}\times\frac{7}{16}\times 2}{\cdot 785\times\frac{13}{16}\times\frac{13}{16}\times 1}=\frac{\cdot 4375\times 2}{\cdot 785\times\cdot 8125}=\frac{1\cdot 37}{1}.$$

The crushing resistance is thus 1·37 times as great as the shearing or the tearing resistance.

To determine (3), the distance of the centre of the rivets from the edge of the plate, we follow actual practice by making this distance one and a half times the diameter of the rivet, i.e. $1·5 \times ·8125 = 1·218$ inches, or, say, $1\frac{1}{4}$ inches.

The design of the required joint is then as follows :—

(1) Diameter of rivets, $\frac{13}{16}$ inch.

(2) Pitch of rivets, $1\frac{3}{4}$ inches.

(3) Distance from centre of rivets to edge of plate, $1\frac{1}{4}$ inches.

90.—(4) The cylindrical barrel of a locomotive boiler is to be 4 feet diameter, and the longitudinal seams are to be double-riveted, butt-jointed, with inside and outside cover straps. The steam pressure is 150 lbs. per square inch. Determine the thickness of the plates and the longitudinal pitch of the rivets.

The thickness of the plates is obtained by using the relation,

$$t = \frac{pd}{2f_t k_t},$$

where p=pressure, in lbs. per square inch,
d=diameter of barrel, in inches,
f_t=allowable tensile stress in plates,
k_t=fractional tearing strength of joint.

In the question, we are not given the value of k_t, so it will be necessary to assume a value.

For a double-riveted butt joint, the tearing efficiency is usually between 70 and 80 per cent., so we may take k_t to be, say, ·75. Assume also that f_t=10,000 lbs. per square inch.

Hence, substituting the known values,

$$t = \frac{150 \times 4 \times 12}{2 \times 10{,}000 \times ·75} = ·48 \text{ inch.}$$

Required thickness of plates=·48 inch, or practically $\frac{1}{2}$ inch.

We must next fix the diameter of the rivets.

Using the rule,

$$D = 1·2\sqrt{t}, \quad \text{where } D = \text{diameter of rivets,}$$
$$t = \text{thickness of plates.}$$

$$D = 1·2\sqrt{\tfrac{1}{2}} = 1·2 \times ·707 = ·85 \text{ inch.}$$

A suitable diameter of rivet would thus be $\frac{13}{16}$ inch.

To determine the longitudinal pitch, we equate the tearing resistance of the plates to the shearing resistance of the rivets. By doing this, we found in the text that for a joint of the type under consideration,

$$P=\frac{\pi D^2 f_s^1}{t f_t^1}+D,$$

where P=longitudinal pitch of rivets,
D=diameter of rivets,
t=thickness of plates,
f_s^1=ultimate shear stress of the rivet material,
f_t^1=ultimate tensile stress of the plate material.

Substituting the known values, and taking

$$\frac{f_s^1}{f_t^1}=\cdot 8,$$

$$P=\frac{\pi\times\frac{13}{16}\times\frac{13}{16}\times\cdot 8}{\frac{1}{2}}+\frac{13}{16}$$

$$=3\cdot 32+\cdot 81=\underline{4\cdot 13 \text{ inches.}}$$

The required pitch is therefore 4·13 inches, or, say, 4 inches.

With this pitch of rivets the joint would be equally strong to resist tearing of the plates or shearing of the rivets, but in actual practice a rather less pitch would probably be adopted in order to make a thoroughly tight joint.

We assumed at the commencement an efficiency of 75 per cent.

The actual efficiency is $\frac{P-D}{P}\times 100,$

$$=\frac{4-\frac{13}{16}}{4}\times 100=\underline{79\cdot 7 \text{ per cent.}}$$

The foregoing example is a typical everyday problem in boiler design. It may be well to point out that, in cases where the boiler is liable to suffer from corrosion, the plates are often made a little thicker than that found to be necessary from the calculation of strength. Thus, in the present example, we found the thickness required to be $\frac{1}{2}$ inch, but some designers would add $\frac{1}{16}$ inch to this, or, in other words, adopt $\frac{9}{16}$-inch plates, for the reasons referred to. Allowance may of course be made by adopting a low working stress in the preliminary calculation.

(5) Design the riveted joint for the longitudinal seams of a Lancashire boiler, the plates of which are $\frac{13}{16}$ inch thick; the joint is to be treble-riveted, butt-jointed, with inside and outside cover straps, the inner strap taking three rows of rivets on each side of the joint, and the outer one two. Assume the strength of the rivets in double shear is only 1·75 times the strength in single shear.

To design the joint, which will be of the form shown in Fig. 38 (*a*), we have to determine (1) the diameter of the rivets; (2) the longitudinal pitch of the rivets; (3) the diagonal pitch; (4) the distance from the centre of the inner rivets to the plate edges; (5) the thickness of the butt straps.

For finding the diameter of the rivets, we use the relation,

$$D=1{\cdot}2\sqrt{t},$$

where D=diameter of rivets,
t=thickness of plates.

We are given that t is $\frac{13}{16}$ inch=·8125 inch.

$$\therefore\ D=1{\cdot}2\sqrt{{\cdot}8125}=1{\cdot}2\times{\cdot}9=\underline{1{\cdot}08\text{ inches.}}$$

For a $\frac{13}{16}$-inch plate, 1 inch rivets are commonly used in boiler work, so we shall adopt this size of rivet for the joint.

Diameter of rivets=$\underline{1\text{ inch.}}$

We have shown in the text dealing with treble-riveted butt joints of the form under consideration that the pitch of the rivets, P, is

$$P=\frac{9\pi D^2 f_s^1}{4tf_t^1}+D.$$

This assumes that the rivets are twice as strong in double as in single shear.

In this particular example, however, we are told that the rivets are to be assumed only 1·75 times as strong in double as in single shear, and we must accordingly modify the above expression for the pitch.

The tearing resistance of the plates is $(P-D)tf_t^1$.

There are four rivets in double shear and one in single shear, and the shearing resistance is then

$$4\frac{\pi}{4}D^2f_s^1 1{\cdot}75+\frac{\pi}{4}D^2f_s^1=2\pi D^2 f_s^1.$$

Equating the tearing and the shearing resistances,

$$(P-D)tf_t^1=2\pi D^2f_s^1$$

$$(P-D)=\frac{2\pi D^2 f_s^1}{tf_t^1}.$$

$$\therefore\ P=\frac{2\pi D^2 f_s^1}{tf_t^1}+D.$$

Substituting the known values, and taking $\frac{f_s}{f_t}={\cdot}8$,

$$P=\frac{2\times\pi\times1^2\times{\cdot}8}{{\cdot}8125}+1=6{\cdot}18+1=\underline{7{\cdot}18\text{ inches.}}$$

We may adopt then a pitch of, say, 7 inches.

Required pitch of rivets=$\underline{7\text{ inches.}}$

This pitch is, of course, the pitch of the rivets in the outer rows, these rivets being spaced just twice as far apart as the rivets in the inner rows. The pitch of the latter rivets will consequently be $3\frac{1}{2}$ inches.

The diagonal pitch should not be less than $\cdot65P+\cdot35D$. The pitch of the inner rows of rivets, which is, of course, the one we are concerned with for the moment, is $3\frac{1}{2}$ inches.

Hence, the diagonal pitch must not be less than

$$\cdot65\times3\cdot5+\cdot35\times1=2\cdot27+\cdot35=2\cdot62 \text{ inches.}$$

A diagonal pitch of $2\frac{5}{8}$ inches may thus be adopted.

Diagonal pitch $=\underline{2\frac{5}{8} \text{ inches.}}$

The distance from the centre of the inner rivets to the plate edges, according to the usual practice, is made equal to $1\frac{1}{2}$D, so in this case it will be $\underline{1\frac{1}{2} \text{ inches.}}$

The thickness of the butt straps, t_s, may be obtained from the relation,

$$t_s=\frac{(P-D)t}{2(P-2D)}.$$

Substituting the known values, $t_s=\frac{(7-1)\frac{13}{16}}{2(7-2\times1)}=\frac{6\times13}{10\times16}=\cdot487$, or nearly $\frac{1}{2}$ inch. This is the theoretical thickness. Actually, the straps would most likely be made $\frac{11}{16}$ inch thick. The general design is, therefore, as follows:—

Thickness of plates . . .	$\frac{13}{16}$ inch.	ANSWERS.
Diameter of rivets . . .	1 ,,	
Longitudinal pitch . . .	7 inches.	
Pitch of inner rows of rivets .	$3\frac{1}{2}$,,	
Diagonal pitch	$2\frac{5}{8}$,,	
Distance to edge of plates from centres of inner rivets . .	$1\frac{1}{2}$,,	
Thickness of butt straps . .	$\frac{11}{16}$ inch.	

Having designed the joint so that the tearing resistance of the plates is equal to the shearing resistance of the rivets, the design may be checked over to ensure that it is also strong enough to resist failure by crushing of the rivets.

The total area of the rivets resisting crushing per pitch is $5Dt$, and the crushing resistance is therefore $5Dtf_c^1=5\times1\times\frac{13}{16}\times f_c^1$.

The tearing resistance $=(P-D)tf_t^1=(7-1)\frac{13}{16}f_t^1$.

Ratio of crushing resistance to tearing resistance

$$=\frac{5\times1\times\frac{13}{16}\times f_c^1}{(7-1)\frac{13}{16}\times f_t^1}=\frac{5\times f_c^1}{6f_t^1}=\frac{5\times1\cdot6}{6\times1}=\frac{8}{6}=1\cdot33.$$

(We are assuming the crushing strength of the rivet material, per square inch, to be equal to $1\cdot6$ times the tensile strength of the plate material.)

The crushing resistance is thus 1·33 times the tearing or the shearing resistance of the joint, and failure by crushing is therefore out of the question.

91.—(6) A simple lap-jointed tie bar is shown in Fig. 40. The tie bar is $4\frac{3}{4}$ inches wide and $\frac{1}{2}$ inch thick, and the rivets are $\frac{7}{8}$ inch diameter. How would this joint fail? Assume the tensile strength of the bar to be 30 tons per square inch, the shearing strength of the rivets 24 tons, and the crushing strength 45 tons per square inch.

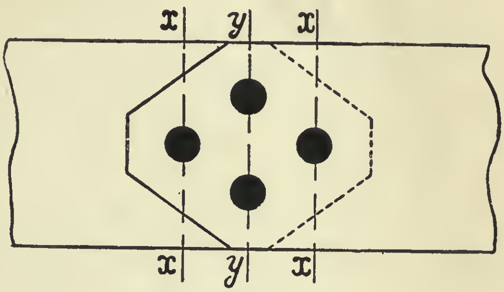

Fig. 40

The joint in question might fail by (1) tearing through the full width of plate at either of the sections *xx*; (2) shearing all the four rivets; (3) tearing through the plate at the section *yy* and shearing the rivet at one of the sections *xx*; (4) by crushing of the rivets. (It should be mentioned that failure is also possible by tearing through the shaped end of the plate along one of the sections *xx*, and shearing the rivets at the other sections, but there is usually an excess of strength to resist failure in this manner, so we shall not consider such failure.)

In order to say in which way failure will take place we must calculate the resistance to failure in each case.

(1) The available metal to resist tearing at the section *xx* is $(b-\text{D})t$, and the tearing resistance is therefore $(b-\text{D})tf_t^1$, where

b=breadth of the bar, in inches,
D=diameter of the rivets, in inches,
t=thickness of the bar, in inches,
f_t^1=tensile strength of the bar material, in tons per square inch.

Substituting the given values,

$$\text{Tearing resistance}=(4·75-·875)\tfrac{1}{2}\times30=58·1 \text{ tons.}$$

(2) There are four rivets in single shear to resist failure of the joint by shearing, so that the total area of rivet section available is $4\frac{\pi}{4}\text{D}^2$.

Total shearing resistance$=4\frac{\pi}{4}\text{D}^2f_s^1=\pi\text{D}^2f_s^1$, where f_s^1=the shearing strength of the rivets. Substituting the known values,

$$\text{Shearing resistance}=\pi\times\tfrac{7}{8}\times\tfrac{7}{8}\times24=\underline{57·75 \text{ tons.}}$$

(3) The metal available to resist tearing at the section yy is $(b-2D)t$, so that the tearing resistance there is $(b-2D)tf_t^1$. Before the joint can fail by tearing at this section, a rivet at one of the sections xx will have to be sheared through. The shearing resistance of this rivet is $\frac{\pi}{4}D^2f_s^1$. The total resistance to failure is the sum of the tearing and the shearing resistances,

$$\therefore \text{ Total resistance}=(b-2D)tf_t^1+\frac{\pi}{4}D^2f_s^1.$$

Substituting the values,

$$\text{Total resistance}=(4{\cdot}75-2\times{\cdot}875)\tfrac{1}{2}\times 30+\frac{\pi}{4}\times\tfrac{7}{8}\times\tfrac{7}{8}\times 24$$
$$=45+14{\cdot}44=\underline{59{\cdot}44 \text{ tons.}}$$

(4) There are four rivets to resist crushing, the available area of rivet metal thus being $4Dt$. The crushing resistance is consequently $4Dtf_c^1$, where f_c^1 is the crushing strength of the metal.

$$\text{Crushing resistance}=4Dtf_c^1=4\times\tfrac{7}{8}\times\tfrac{1}{2}\times 45$$
$$=\underline{78{\cdot}75 \text{ tons.}}$$

Summing up,

(1) Tearing resistance	$=58{\cdot}1$ tons.
(2) Shearing resistance	$=57{\cdot}75$,,
(3) Combined tearing and shearing resistance	$=59{\cdot}44$,,
(4) Crushing resistance	$=78{\cdot}75$,,

Reviewing these results, we see that the resistances for (1), (2) and (3) are almost the same, showing that the joint has been well designed, as it would be almost as liable to fail in one way as the other. The shearing resistance is slightly lower than the others, and the joint would consequently fail by shearing of the rivets. The calculation of the crushing resistance shows that the joint is amply safe as regards crushing of the rivets.

(7) Two lengths of a flat steel tie bar are to be connected together by a butt joint with double straps. The tie is to carry a load of 45 tons and is $\frac{3}{4}$ inch thick. Determine the diameter and the number of the rivets required.

The diameter of the rivets may be determined from the relation,

$$D=1{\cdot}1\sqrt{t},$$

where D=diameter of rivets and t the thickness of the tie bar. We are given that t is $\frac{3}{4}$ inch, therefore,

$$D=1{\cdot}1\sqrt{{\cdot}75}={\cdot}95 \text{ inch.}$$

In actual practice, $\frac{7}{8}$-inch rivets would probably be used.

$$\text{Diameter of rivets}=\underline{\tfrac{7}{8} \text{ inch.}}$$

We may assume the rivets to be so arranged that there will be one rivet at each of the two end rivet sections of the joint, two at each of the sections next to the end sections, then three, and so on. (See Fig. 39.)

The strength of the bar to resist tearing through either of the sections where there is only a single rivet is $(b-D)tf^1$, where

b=breadth of bar, in inches,
D=diameter of rivets, in inches,
t=thickness of bar, in inches,
f_t^1=tensile strength of bar material, in lbs. per square inch.

We are not told in the question what the breadth of the bar is, and before we can proceed with the problem we shall have to calculate this.

The cross sectional area of the tie at either of the end rivet sections is $(b-D)t=(b-\frac{7}{8})\frac{3}{4}$ square inches. The load on the tie is 45 tons, so if we assume a safe working stress of 5 tons per square inch, the sectional area required will be $\frac{45}{5}=9$ square inches. Hence,

$$(b-\tfrac{7}{8})\tfrac{3}{4}=9,$$

from which $b=12{\cdot}88$, or, say, $\underline{13 \text{ inches.}}$

Breadth of tie bar = $\underline{13 \text{ inches.}}$

Taking the ultimate tensile strength of the metal to be 30 tons per square inch, the resistance of the bar to tearing through either of the sections where there is only one rivet is $(b-D)tf_t^1=(13-\frac{7}{8})\frac{3}{4}\times 30=272{\cdot}8$ tons.

As the joint is a double butt joint, all the rivets will be in double shear, and the shearing strength of each rivet is $\frac{\pi}{4}D^2f_s^1 2=\frac{\pi}{2}D^2f_s^1$, where f_s^1=the shearing strength of the rivet metal. Taking f_s^1 to be 24 tons per square inch, the shearing resistance of each rivet is $\frac{\pi}{2}\times\frac{7}{8}\times\frac{7}{8}\times 24=28{\cdot}9$ tons.

The number of rivets should be such that the joint will be equally liable to fail by tearing through the bar or shearing of the rivets.

Tearing resistance of bar $=272{\cdot}8$ tons.
Shearing resistance of each rivet $=28{\cdot}9$,,

Therefore,

$$\text{Number of rivets required}=\frac{272{\cdot}8}{28{\cdot}9}=\underline{9{\cdot}45}, \text{ or, say, } \underline{10.}$$

With 10 rivets, the crushing resistance of the joint, assuming an ultimate crushing stress of, say, 48 tons per square inch, will be $10 \times \frac{7}{8} \times \frac{3}{4} \times 48 = 315$ tons, which is greater than the tearing resistance.

$$\left.\begin{array}{ll} \text{Breadth of tie bar required} & = 13 \text{ inches.} \\ \text{Diameter of rivets} & = \frac{7}{8} \text{ inch.} \\ \text{Number of rivets} & = 10. \end{array}\right\} \text{ANSWERS.}$$

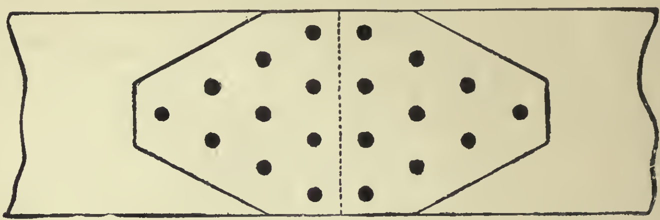

Fig. 41

The sketch, Fig. 41, shows how the rivets may be conveniently arranged.

BOARD OF TRADE REQUIREMENTS CONCERNING MARINE BOILERS

92.—The Board of Trade, in their "Instructions as to the Survey of Passenger Steamships," lay down certain rules and regulations which must be adhered to before the ship is granted her passenger certificate. The Board have their engineer surveyors, who watch the progress of the work from the time the plates are made right up to the time the ship is finished. The surveyors inspect the plates both for the ship's structure and for the boilers; they witness the tests on specimens cut off the different plates, examine the workmanship during construction, and, in the case of the boilers, witness the hydraulic test on completion.

The following is an extract from the above-mentioned Instructions, relating to the testing of boiler plates :—

93.—**Number and Nature of Tests.** A tensile and a bending test should be taken from each plate, as rolled; but when the weight exceeds two and a half tons, a tensile and a bending test should be taken from each end. If the plates are not to be subjected to a greater stress than is allowed for iron, only bending tests are necessary.

The plates for manhole doors, and for compensating rings around the openings for doors, should be tested in the usual manner.

Plates.—The tensile strength of plates not intended to be worked in the fire or exposed to flame should be between 27 and

32 tons per square inch, and that of other plates 26 to 30 tons per square inch. The elongation should not be less than 20 per cent. in a length of 8 inches for material $\frac{3}{8}$ inch in thickness and upwards, which is required to have a tensile strength of 27 to 32 tons per square inch, and not less than 23 per cent. if the tensile strength is required to be between 26 and 30 tons per square inch. For material under $\frac{3}{8}$ inch in thickness, the elongation may be reduced; but, for each eighth of an inch of diminution in thickness, the reduction should not be more than 3 per cent. below the elongations mentioned.

Bend Tests for Plates.—Bending test pieces should withstand being bent, without fracture, until the sides are parallel at a distance apart of not more than three times the thickness of the specimen. The bending tests of the plates not intended to be worked in the fire or exposed to flame may be made with strips in the same condition as the plates; those from other plates should be made with strips which have been tempered.

Tensile Strength and Elongation of Stays, Angles, and Tee Bars.—The tensile strength of longitudinal stays, angles, and tee bars should be between 27 and 32 tons per square inch, with an elongation of not less than 20 per cent. measured on the appropriate test piece. (The gauge length of these test pieces for a rectangular bar is 8 inches, and for a round bar it is not less than eight times the diameter, or for test pieces over 1 inch in diameter not less than four times the diameter.)

For bars for combustion chamber stays, the tensile strength should be between 26 and 32 tons per square inch, with an elongation of not less than 23 per cent. measured on the standard test piece. When, however, stay bars are tested on a gauge length of four times the diameter, the elongations should be 24 per cent. and 28 per cent. respectively.

For tee or angle bars under $\frac{3}{8}$ inch in thickness the elongation may be 3 per cent. below that specified for plates.

Bend Tests for Stays, etc.—Bending test pieces should withstand being bent, without fracture, until the sides are parallel at a distance apart of not more than three times the thickness or diameter of the specimen.

Rivet Bars.—The tensile strength of rivet bars should be between 26 and 30 tons per square inch, with an elongation of not less than 25 per cent. if measured on a gauge length of eight times the diameter, or 30 per cent. if measured on a gauge length of four times the diameter—the latter gauge length being only used when the specimen is over 1 inch in diameter.

Rivets.—The rivets are subjected to the following test:—

(*a*) The rivet shanks to be bent cold and hammered until the two parts of the shank touch, without fracture, on the outside of the bend. (*b*) The rivet heads to be flattened, while hot, until their diameter is two and a half times the diameter of the shank, without cracking at the edges.

Tubes.—(*a*) Solid-drawn steel steam pipes, boiler tubes, etc., subject to internal pressure. The tensile strength should range between 23 and 30 tons per square inch, and the elongation should not be less than 20 per cent. in a length of 8 inches, or 18 per cent. if the thickness of the tubes is less than $\frac{1}{4}$ of an inch.

(*b*) Solid-drawn steel tubes subject to external pressure. The tensile strength should range between 23 and 30 tons per square inch, and the elongation should be at least that required for similar solid-drawn steam pipes.

(*c*) Steel lap-welded tubes subject to external pressure. Tensile and bending tests should be made from 25 per cent. of the strips from which the tubes are made. The tensile strength should range between 23 and 30 tons per square inch, and the elongation should be at least 20 per cent. in a length of 8 inches when the strips are tested in their normal condition.

94.—**Steel Boilers.** The thickness of plates other than tube strips used in the construction of boilers should not be less than $\frac{5}{16}$ inch.

It is expected that the rivet holes will be drilled, and not punched. Plates that are drilled in place should be taken apart and the burr taken off, and the holes slightly countersunk from the outside.

Butt straps should be cut from plates and not from bars. Steel plates which have been welded should not be passed if subject to a tensile stress, and those welded and subject to a compressive stress should be efficiently annealed.

Local heating of the plates should be avoided, as many plates have failed from having been so treated. All plates that have been flanged or locally heated, and all stays and stay tubes which have been locally heated, should be carefully annealed after being so treated.

Cylindrical Boiler Shells.—The Board of Trade consider that boilers well constructed, well designed, and made of good material should be allowed an advantage, in the matter of working pressure, over boilers inferior in any of the above respects, as, unless this is done, the superior boiler is placed at a disadvantage, and good workmanship and material will be discouraged. They have therefore caused the following rules to be prepared :—

When the cylindrical shells of boilers are made of material which has been duly tested and approved, with all the rivet holes drilled in place and all the seams fitted with double butt straps, each of at least five-eighths the thickness of the plates they cover, and all the seams at least double-riveted with rivets having an allowance of not more than 87·5 per cent. over the single shear, provided that the boilers have been open to inspection during the whole period of construction, then 4·5 may be used as the factor of safety, the minimum actual tensile strength of the plates being used in calculating the working pressure.

When the above conditions are not complied with, the additions in the following scale should be made to the factor of safety, according to the circumstances of each case:—

Longitudinal Seams.	A†	·15	To be added if all the holes are fair and good, but drilled out of place after bending.
	B†	·3	To be added if all the holes are fair and good, but drilled before bending.
	C	·2	To be added if double butt straps are not fitted, and the seams are lapped and double-riveted.
	D	·1	To be added if double butt straps are not fitted, and the seams are lapped and treble-riveted.
	E	·3	To be added if only single butt straps are fitted, and the seams are double-riveted.
	F	·15	To be added if only single butt straps are fitted, and the seams are treble-riveted.
	G	1·0	To be added if any joint is single-riveted.
	H*	·4	To be added if there are two or more belts of plates, and the seams are not properly crossed.
Circumferential Seams.	I†	·1	To be added if the holes are fair and good, but drilled out of place after bending.
	J†	·15	To be added if the holes are fair and good, but drilled before bending.
	K	·1	To be added if the seams are fitted with single butt straps and are double-riveted.
	L	·2	To be added if the seams are fitted with single butt straps and are single-riveted.
	M	·1	To be added if the seams are fitted with double butt straps and are single-riveted.
	N	·1	To be added if the seams are lapped and double-riveted.
	O	·2	To be added if the seams are lapped and single-riveted.
	P‡	·3	To be added if the boiler is of such a length as to fire from both ends, or is of unusual length, as in the case of flue boilers, and the seams are fitted as described opposite K, M, or N; but if the seams are as described opposite L or O, P·4 should be added—in lieu of the addition designated by K, M, N, L, or O, as the case may be.

* The factor may be increased still further if the workmanship is such as, in the surveyor's judgment, to render such increase necessary.

† If the holes are to be rimered or bored out in place, the case should be submitted to the Board as to the reduction or omission of A, B, I and J, as heretofore.

‡ If the middle circumferential seams are double-strapped and double-riveted, or lapped and treble-riveted, and the calculated strength is not less than 65 per cent. of the solid plate, no addition to the factor need be made in respect of the length of the boiler.

95.—**Board of Trade Boiler Formulæ.** Boiler pressure (neglecting seams) $= \frac{2tf_t^1}{d \times F}$,

where t = thickness of plate,
f_t^1 = tensile strength of plate, as obtained from test—this is usually about 28 tons per square inch for steel,
d = diameter of boiler, in inches,
F = factor of safety.

The bursting pressure of a cylindrical vessel subject to internal pressure has been proved (see page 85) to be $\frac{2tf_t^1}{d}$. The safe working pressure is obtained by dividing this by the factor of safety, F.

The amount of metal left in the plate after drilling the rivet holes is $(p-d)t$, where p = the pitch of rivets, d = diameter of rivets, and t = the thickness of plate. If f_t^1 is the tensile strength of the material, the strength of the material after drilling = $(p-d)t \times f_t^1$. The strength before drilling is evidently $p \times t \times f_t^1$. Therefore, the ratio of the strengths after and before drilling is $\frac{(p-d)t \times f_t^1}{p \times t \times f_t^1}$. Cancelling, we get the ratio, $\frac{p-d}{p}$, as the proportionate strength of the plate after drilling; this, expressed as a percentage, is $\frac{p-d}{p} \times 100$, which is the percentage strength of plate at the joint as compared with the solid plate.

Let n = number of rivets in a pitch.
A = area of rivet.
p = pitch of rivets.
t = thickness of plates.
f_s^1 = shearing strength of rivets.
f_t^1 = tensile strength of plates.

As before, the strength of the plate before drilling = $p \times t \times f_t^1$, and the strength of riveting section = $A \times n \times f_s^1$, so that the ratio of the strength of the rivets to the strength of the solid plate is $\frac{A \times n \times f_s^1}{p \times t \times f_t^1}$, the percentage strength being $\frac{A \times n \times f_s^1}{p \times t \times f_t^1} \times 100$. This formula is used when the rivets are in single shear; when they are in double shear the Board of Trade allow seven-eighths more, that is to say, they assume the rivets to be seven-eighths stronger when in double shear.

The usual value for f_s^1 is 23 tons per square inch and for f_t^1 it is 28 tons per square inch; substituting these values we have the complete formula for the percentage strength of the rivets when in double shear as $\frac{A\times n}{p\times t}\times\frac{23}{28}\times\frac{15}{8}\times 100.$

When designing a joint for a boiler it is necessary to make the strength of the plate section left after drilling as near as possible equal to the strength of the rivet section. It would be absurd,

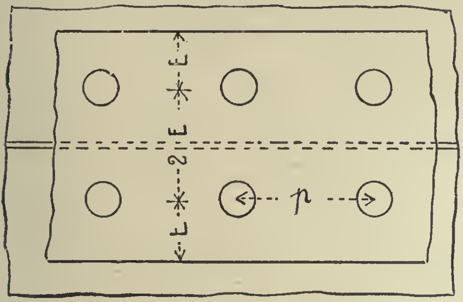

Fig. 42

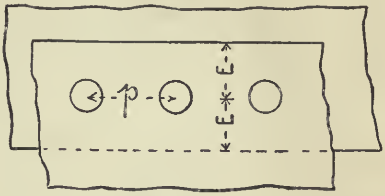

Fig. 43

for instance, to have a riveting section equal to 90 per cent. of the solid plate, and the strength of the plate after drilling only 50 per cent. of the solid plate.

Working out both these percentages, we select the weaker for use in the formula for the safe working pressure of the boiler.

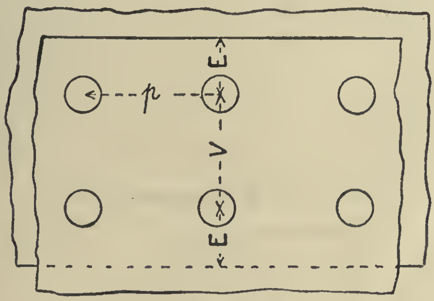

Fig. 44

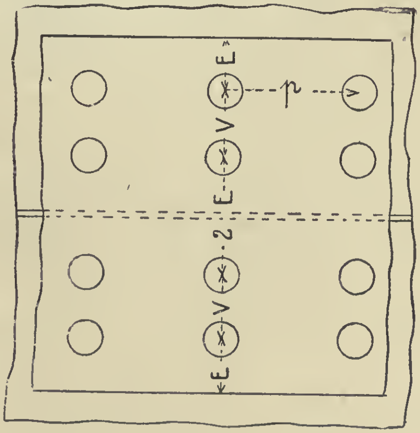

Fig. 45

It has been explained above that the working pressure, neglecting seams, is equal to $\frac{2tf_t^1}{d\times F}$.

If we multiply this by the percentage strength of the riveting section, or the percentage strength of the drilled plate, using whichever value is the lesser, we obtain the formula, Safe working pressure, in lbs. per square inch,

$$=\frac{2tf_t^1 \times \text{percentage strength of joint}}{d \times F}.$$

Note that if the tensile strength of the plate is given in tons

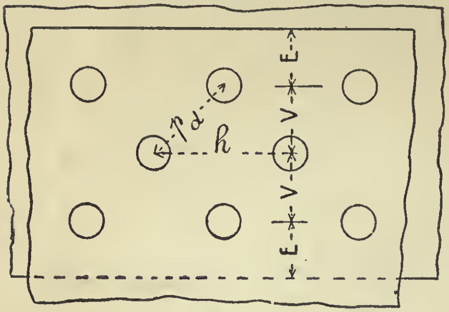

Fig. 46

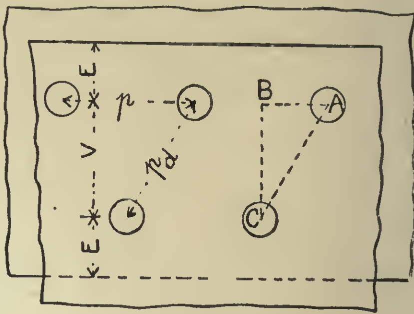

Fig. 47

per square inch this must be multiplied by 2240 to convert the answer to lbs. per square inch.

96.—**Riveting.** According to the Board of Trade rules, the distances between the rows of rivets (called V in Figs. 44 to 50), and the distance between the rivet and the edge of the plate (E), is in all these figures, $\frac{3 \times d}{2}$

Chain-riveted joints (Figs. 44 and 45) V=not less than $2 \times d$.

Diagonal pitch (Figs. 46, 47, 48 and 49) $p_d = \frac{6p+4d}{10}$.

This, of course, also fixes the value of V in these joints, because in Fig. 47, looking on ABC as a right-angled triangle,

$$BC^2 = AC^2 - AB^2.$$

But $BC = V$, $AB = \frac{p}{2}$, and $AC = p_d$.

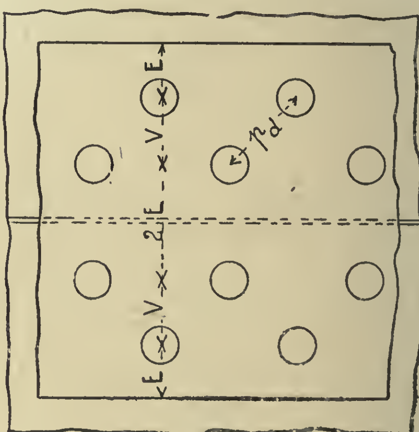

Fig. 48

Then, $$V^2 = \left(\frac{6p+4d}{10}\right)^2 - \left(\frac{p}{2}\right)^2$$

$$= \left(\frac{6p+4d}{10} + \frac{p}{2}\right)\left(\frac{6p+4d}{10} - \frac{p}{2}\right)$$

$$= \left(\frac{6p+4d+5p}{10}\right)\left(\frac{6p+4d-5p}{10}\right).$$

$$V = \sqrt{\frac{(11p+4d)(p+4d)}{10^2}}$$

$$= \frac{\sqrt{(11p+4d)(p+4d)}}{10}.$$

Chain and zig-zag riveted joints in which every alternate rivet is omitted from the outer row or from the outer and inner rows,

$$E=\frac{3\times d}{2} \text{ in all cases.}$$

Chain-riveted joints,

$$\left.\begin{array}{ll} & V=\dfrac{\sqrt{(11p+4d)(p+4d)}}{10} \\ \text{or,} & V=2\times d. \end{array}\right\} \text{The greater of these two values of V to be used.}$$

Diagonal pitch between outer and middle rows for zig-zag riveted joints of type shown in Fig. 50, $p_d=\frac{3}{10}p+d$.

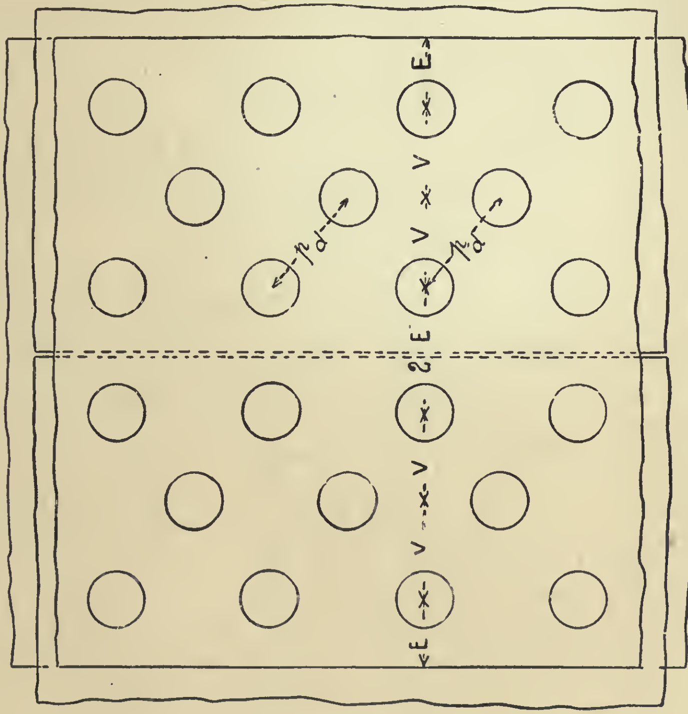

Fig. 49

V for these joints is found in the same way as it was for the joints of Figs. 46, 47, 48, and 49. $V=\sqrt{(\frac{11}{20}p+d)(\frac{1}{20}p+d)}$.

Diagonal pitch between inner and middle rows for joint shown in Fig. 50,

$$p_d=\frac{3p+4d}{10},$$

from which, $$V_1=\frac{\sqrt{(11p+8d)(p+8d)}}{20}$$

Note.—The *minimum* value of V or V_1 for chain-riveted joints is given as $2d$, but $\frac{4d+1}{2}$ is more desirable.

Maximum Pitches for Riveted Joints.

t=thickness of plate, in inches,
p=maximum pitch of rivets, in inches—provided it does not exceed $10\frac{1}{2}$ inches; and
C=constant applicable from following table.

Number of rivets in one pitch.	Constants for lap joints.	Constants for double butt strap joints.
1	1·31	1·75
2	2·62	3·50
3	3·47	4·63
4	4·14	5·52
5	..	6·0

Then, $(C \times t) + 1\frac{5}{8} = p.$

The maximum pitch should not exceed $10\frac{1}{2}$ inches with the thickest plates for boiler shells.

97.—**Openings in Shells.** The openings in the shells of cylindrical boilers should have their shorter axes placed longitudinally.

Compensating rings of at least the same effective sectional area as the plates cut out, and not less in thickness than the plates to which they are attached, should be fitted around all manholes and openings, or the surrounding portion of the plates otherwise efficiently stiffened. When a ring is not fitted around such an opening, and the plate is flanged for compensation, the total depth, D, of the flange should not be less than that given by the following equation:—

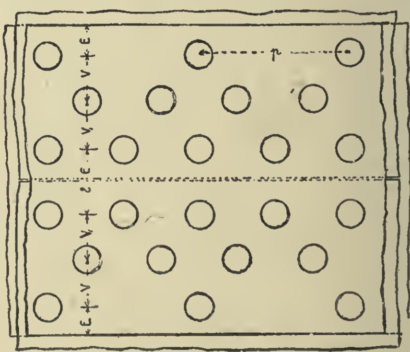

Fig. 50

$$D = \sqrt{\text{width of opening} \times \text{thickness of plate}}.$$

Solid steel stays may be allowed a working stress of 9000 lbs. per square inch of net section. Solid iron stays are allowed a stress of 7000 lbs. per square inch of net section, but if the stays have been welded, then only 5000 lbs. per square inch is allowed. Steel stays which have been welded should not be passed. (This does not apply to stay tubes which are welded longitudinally.)

Note.—If the iron from which the stays are made has been tested by a Board of Trade surveyor, and found to have a tensile strength of $21\frac{1}{2}$ tons per square inch, and an elongation, measured on a length of 8 inches, of not less than 27 per cent., then a stress of 9000 lbs. per square inch is allowed on the stays if solid and unwelded.

Stay Tubes.—A stress of 7500 lbs. per square inch of net section is allowed on steel tubes, and a stress of 6000 lbs. per square inch on iron, provided, in both cases, their net thickness is never less than $\frac{1}{4}$ inch.

Ordinary Tubes.—The thickness of ordinary smoke tubes should not be less than that found by the following formula :—

$$T=\frac{B\times D}{9000}+\cdot085,$$

where T=thickness of tube, in inches,
D=outside diameter of tube, in inches,
B=working pressure.

98.—**Flat Surfaces.** The pressure on plates forming flat surfaces is found by the following formula :—

$$\frac{C(\text{or } c)\times(T+1)^2}{S-6}=\text{working pressure},$$

where T=thickness of plate, in sixteenths of an inch,
S=surface supported, in square inches,
C=constant for steel } according to the following cir-
c=constant for iron } cumstances.

C=240 *c*=192	When the plates are not exposed to the impact of heat or flame, and the stays are fitted with nuts on both sides of the plates, and doubling strips not less in width than two-thirds the pitch of the stays, and of the thickness of the plates, are securely riveted to the outside of the plates they cover.
C=210 *c*=168	When the plates are not exposed to the impact of heat or flame, and the stays are fitted with nuts on both sides of the plates, and with washers not less in diameter than two-thirds the pitch of the stays, and of the same thickness as the plates, securely riveted to the outside of the plates they cover.
C=165 *c*=132	When the plates are not exposed to the impact of heat or flame, and the stays are fitted with nuts on both sides of the plates, and with washers outside the plates at least three times the diameter of the stay and two-thirds the thickness of the plates they cover.
C=150 *c*=120	When the plates are not exposed to the impact of heat or flame, and the stays are fitted with nuts on both sides of the plates.

$C=112{\cdot}5$ $c=90$	When the tube plates are not exposed to the direct impact of heat or flame, and the stays are fitted with nuts.
$C=77$ $c=70$	When the tube plates are not exposed to the direct impact of heat or flame, and the stay tubes are screwed into the plates and expanded.
$C=77$ $c=70$	When the plates are not exposed to the impact of heat or flame, and the stays are screwed into the plates and riveted over.
$C=75$ $c=60$	When the plates are exposed to the impact of heat, with steam in contact with the plates, and the stays fitted with nuts and washers, the latter being at least three times the diameter of the stay and two-thirds the thickness of the plates they cover.
$C=67{\cdot}5$ $c=54$	When the plates are exposed to the impact of heat, with steam in contact with the plates, and the stays fitted with nuts only.
$C=100$ $c=80$	When the plates are exposed to the impact of heat or flame, with water in contact with the plates, and the stays screwed into the plates and fitted with nuts.
$C=66$ $c=60$	When the plates are exposed to the impact of heat or flame, with water in contact with the plates, and the stays screwed into the plates and having the ends riveted over to form substantial heads.
$C=39{\cdot}6$ $c=36$	When the plates are exposed to the impact of heat, with steam in contact with the plates, with the stays screwed into the plates and having the ends riveted over to form substantial heads.

In calculating the working pressure of the portion of tube plates between the boxes of tubes, the value of S in the formula should be found as follows :—

$$\frac{D^2+d^2}{2}=S,$$

where D=horizontal pitch of the stay tubes, in inches,
d=vertical ,, ,, ,,

The pitches should be measured from centre to centre of the stay tubes, and no deduction should be made for any tubes in the contained surface.

In the body of tube plates, the value of S may be found in the ordinary way, and the area of the tubes in the space bounded by the stay tubes may be deducted.

99.—**Compressive Stress on Tube Plates.** The Board of Trade formula is :—

$$\frac{(D-d)T\times 28000}{W\times D}=p,$$

where D = least horizontal distance between centres of tubes, in inches,
d = inside diameter of ordinary tubes, in inches,
T = thickness of tube plate, in inches,
W = width of combustion box, in inches.

If we consider the combustion chamber top, it is evident that the weight thereon, due to the pressure of the steam, is taken by the stays in the combustion chamber girder stay. This girder stay is then like a beam, and the total weight is transferred to its supports—in this case, the back tube plate and the combustion chamber back, each of which takes half the weight.

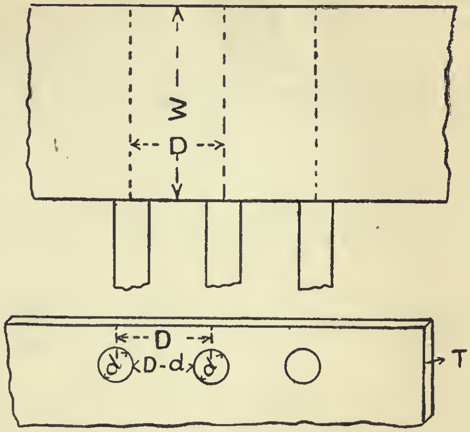

Fig. 51

Consider a portion of the combustion chamber top, the area of which is D×W. (See Fig. 51.) If p is the boiler pressure, the load on this area is $D \times W \times p$. As explained above, half of this load is taken by the tube plate and half by the combustion chamber back. But the tube plate has had holes drilled in it for the tubes, and the metal left to resist this load is $(D-d)T$ where d is the *internal* diameter of the tubes. (See Fig. 51.) If f_c is the compressive stress allowed in the material, the strength of the tube plate to resist this load is $(D-d)T \times f_c$, and this resistance must be equal to the load put upon it. Therefore,

$$(D-d)T \times f_c = \frac{D \times W \times p}{2}.$$

$$\therefore\ p = \frac{2(D-d)T \times f_c}{W \times D}.$$

If we compare this with the Board of Trade formula,

$$p = \frac{(D-d)T \times 28{,}000}{W \times D},$$

we see that the constant 28,000 takes the place of $2f_c$, from which, $2f_c = 28{,}000$.

$$\therefore\ f_c = \frac{28{,}000}{2} = 14{,}000 \text{ lbs. per square inch,}$$

which is the compressive stress allowed on the tube plates.

Notice that for an iron tube plate the Board of Trade say a constant of 22,000 should be used, which shows that the compressive stress for iron is fixed at 11,000 lbs. per square inch.

100.—**Combustion Chamber Girders.** The Board of Trade formula is

$$\frac{C\times d^2\times T}{(W-P)D\times L}=p,$$

where W=width of combustion chamber, in inches,
P=pitch of supporting stays, in inches,
D=distance between the girders from centre to centre, in inches,
L=length of girder, in feet,
d=depth of girder, in inches,
T=thickness of girder, in inches,
N=number of supporting stays,

$C=\frac{N\times 1320}{N+1}$, when the number of stays is odd,

$C=\frac{(N+1)1320}{N+2}$, when the number of stays is even.

Note.—The constant 1320 applies to steel; when iron is used, this constant is reduced to 1200.

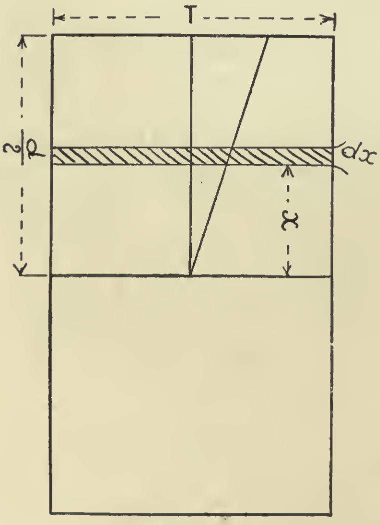

Fig. 52

The moment of resistance of a rectangular beam is $\frac{Td^2S}{6}$, where

T=thickness of beam,
d=depth of beam,
S=stress allowed.

Take a strip of the beam distant x inches from the neutral axis (Fig. 52) and of thickness dx. The area of this strip is Tdx. The stress varies directly as the distance from the neutral axis, and therefore the stress at x inches from the neutral axis is $\frac{x}{\frac{d}{2}}$ of the stress at the outside edge.

The load on the strip $=\frac{T\times x\times 2Sdx}{d}$, and the moment about the neutral axis$=\frac{T\times x\times 2S\times x}{d}dx$. Taking the sum of all such moments as these between the limits, $x=0$ and $x=\frac{d}{2}$, we obtain the moment of resistance for one-half the beam.

Now, $$\int_0^{\frac{d}{2}} \frac{2TSx^2}{d} dx = \frac{2TS}{d} \int_0^{\frac{d}{2}} x^2 dx = \frac{2TS}{d}\left[\frac{x^3}{3}\right]_0^{\frac{d}{2}} = \frac{2TS}{d} \times \frac{d^3}{8\times3} = \frac{Td^2S}{12}.$$

Therefore, the moment of resistance for the whole beam is

$$\frac{Td^2S}{12} \times 2 = \frac{Td^2S}{6}.$$

Consider a combustion chamber girder with two stays. (See Fig. 53.)

The actual surface to be supported by the stays is (W−P), because only flat surfaces require staying, and the radii of the flanges for the front and back plates are approximately half the pitch. If p is the pressure of the steam and D the distance in inches to the next girder, the load to be supported by the stays is (W−P)Dp.

The stays transfer the load from the combustion chamber crown to the girder.

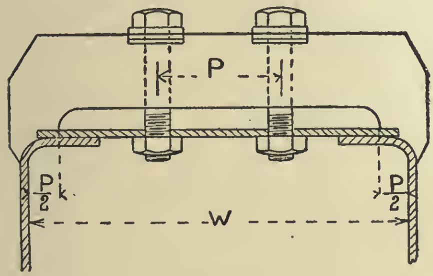

Fig. 53

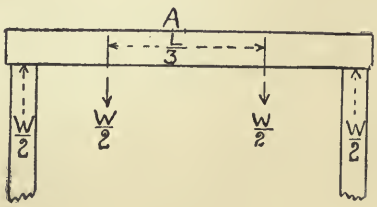

Fig. 54

In Fig. 54, we have shown diagrammatically the effect of two stays in the girder. Notice each stay brings a load on the girder equal to $\frac{W}{2}$. If there were three stays, each would have $\frac{W}{3}$; if four $\frac{W}{4}$, and so on. In each case the reaction at the supports, viz. at the tube plate and the combustion chamber back plate, is $\frac{W}{2}$.

Referring to Fig. 54, take moments about the centre of the beam A. The moment tending to turn the beam in a clockwise direction is $\frac{W}{2}\times\frac{L}{2}$, where L is the length of the beam; and the moment tending to turn it in a counter-clockwise direction is $\frac{W}{2}\times\frac{L}{6}$. Therefore, the bending moment is

$$\frac{W}{2}\times\frac{L}{2} - \frac{W}{2}\times\frac{L}{6} = \frac{WL}{4} - \frac{WL}{12} = \frac{3WL-WL}{12} = \frac{2WL}{12} = \frac{WL}{6} \text{ pound-inches.}$$

K

Now the bending moment must equal the moment of resistance of the beam. We have already seen that W, the load carried, is $(W-P)Dp$. Substituting this value for W in the bending moment and equating it to the moment of resistance we have

$$\frac{(W-P)Dp\times L\text{ inches}}{6}=\frac{STd^2}{6}.$$

In the Board of Trade formula, L is given in feet, which is equivalent to 12L inches.

$$\therefore\ \frac{(W-P)Dp\times 12L}{6}=\frac{STd^2}{6}.$$

$$\therefore\ p=\frac{STd^2}{(W-P)D\times 12L}.$$

Comparing this with the Board of Trade formula we have $\frac{S}{12}$ for C, and C for two bolts $=\frac{2+1\times 1320}{2+2}=990$.

Therefore, $\frac{S}{12}=990$, and $S=11{,}880$.

$\therefore$ Stress allowed in steel girders=11,880 lbs.

The student should work out for himself the case of three stays or four stays; he will find in each case that the stress allowed is 11,880 lbs.

101.—**Plain Furnaces.** As explained on p. 88, Sir William Fairbairn deduced the following empirical formula for the collapsing pressure of a plain furnace:—

$$\frac{806{,}300\times t^{2\cdot 19}}{L\times D},$$

where t=thickness of the plate, in inches;
L=length of furnace, in feet;
D=diameter of furnace, in inches.

Lloyd's take a factor of safety of 9, use t^2 in place of $t^{2\cdot 19}$, and use the following formula as the safe working pressure:—

$$\text{Working pressure}=\frac{89{,}600t^2}{L\text{ feet}\times D\text{ inches}}.$$

The Board of Trade convert this 89,600 into round numbers of 90,000, and to compensate for this they add one on to the length, giving the formula,

$$\frac{90{,}000\times t^2}{(L+1)\times D}=p.$$

The length is measured in feet, and diameter and thickness in inches.

This is the formula for iron furnaces, and the Board allow an increase of 10 per cent. on the constant for steel; thus, the formula for a plain steel furnace is

$$p=\frac{99{,}000\times t^2}{(L+1)\times D},$$

provided it does not exceed that found by the following formula:—

$$\frac{{}^{*}9900\times\text{thickness in inches}}{\text{diameter in inches}}=p.$$

The Board also state that the second formula limits the crushing stress on the material to 4950 lbs. per square inch. If we use the ordinary expression for the internal pressure which may be carried safely by a thin cylindrical vessel, viz. $\frac{2tS}{D}$, where S is the allowable stress, and equate it to this formula, thus:—

$$\frac{2tS}{D}=\frac{9900t}{D},$$

we get $S=4950$ lbs. per square inch.

102.—**Corrugated Furnaces.** The Board of Trade formula for corrugated furnaces of the Morison, Fox, or Deighton types is

$$\frac{14{,}000T}{D}=\text{working pressure.}$$

In the Fox furnace, the pitch of the corrugations should not exceed 6 inches, and in the Morison and Deighton furnaces the pitch should not exceed 8 inches; also, the depth from the top of the corrugations outside to the bottom of the corrugations inside should not be less than 2 inches.

The diameter D is measured at the bottom of the corrugations to the outside of the plates, that is, it is the least outside diameter.

In the Morison suspension bulb type of furnace the working pressure is obtained from the following formula:—

$$\frac{15{,}000T}{D}=\text{working pressure,}$$

where T=thickness of the plain parts between the bulbs, in inches;
D=outside diameter at the middle of the plain parts between the bulbs, in inches.

In each of these types of furnaces the plain parts at the back ends should be so made that the length, measured from the water side of the back tube plate to the centre line of the back end corrugation, does not exceed 9 inches. Also, the distance at the front end, measured from the centre of the rivets by which the furnace is secured to the front end plate, to the centre of the first corrugation, should not exceed 9 inches.

* For an iron furnace the constant is 9000.

Worked Examples

103.—(1) Thickness of plates is $1\frac{1}{4}$ inches. Pitch of rivets 6 inches. Joint is to be treble-riveted with double butt straps. Find the diameter of the rivets to give equal strength to the plate. Ultimate strength of rivets, 23 tons per square inch, and ultimate strength of plates, 28 tons per square inch.

$$\frac{p-d}{p}=\frac{\text{A}\times n}{p\times t}\times\frac{23}{28}\times\frac{15}{8}.$$

where
p=pitch of rivets;
d=diameter of rivets;
A=area of rivets;
n=number of rivets;
t=thickness of plate.

Multiplying both sides of the equation by p,

$$p-d=\frac{\text{A}\times n}{t}\times\frac{23}{28}\times\frac{15}{8}.$$

Now $p=6$; $\text{A}=d^2\times\frac{\pi}{4}$; $n=3$; $t=1\frac{1}{4}$; d is unknown.

$$\therefore\ 6-d=\frac{d^2\times\cdot7854\times23\times15\times4\times3}{5\times28\times8}.$$

i.e.
$$6-d=2\cdot9d^2.$$
$$\therefore\ 2\cdot9d^2+d=6.$$

Divide by 2·9,
$$d^2+\cdot345d=2\cdot07,$$
$$d^2+\cdot345d+\left(\frac{\cdot345}{2}\right)^2=2\cdot07+(\cdot1725)^2,$$
$$(d+\cdot1725)^2=+2\cdot0997,$$
$$d+\cdot1725=\pm\sqrt{2\cdot0997},$$
$$d=1\cdot449-\cdot1725,$$
$$d=1\cdot2765.$$

This would be given as $1\frac{1}{4}$ inches diameter. Answer.

(2) Design a treble-riveted double butt strap joint with alternate rivets omitted in the outer rows; the boiler plates are $1\frac{1}{4}$ inches thick, and the pitch of rivets is not to exceed 10 inches. The efficiency of the joint to be as high as practicable.

As the pitch is not to exceed 10 inches, we shall start by assuming the maximum pitch, 10 inches. Equating the strength of the plate to the riveting strength,

$$\frac{p-d}{p}=\frac{A\times n}{p\times t}\times\frac{23}{28}\times\frac{15}{8}.$$

The pitch we have taken as 10 inches,

$$\therefore \quad 10-d=\frac{d^2\times\cdot7854\times5\times23\times15\times4}{5\times28\times8}.$$

Note that in a joint of this description, there are five rivets in a pitch. (See Fig. 50.) Then,

$$10-d=4\cdot84d^2.$$
$$4\cdot84d^2+d-10=0.$$

Solving this quadratic,

$d=1\cdot33$ inches, say, $\underline{1\tfrac{5}{16}\text{ inches, diameter of rivets.}}$

$$\text{Efficiency of plate}=\frac{p-d}{p}=\frac{10-1\frac{5}{16}}{10}=\frac{8\cdot6875}{10}=\underline{\cdot868.}$$

$$\text{Efficiency of rivets}=\frac{\text{A}\times n}{p\times t}\times\frac{23}{28}\times\frac{15}{8}$$

$$=\frac{21\times21\times\cdot7854\times5\times23\times15\times4}{16\times16\times5\times10\times28\times8}=\underline{\cdot833.}$$

On looking at Fig. 55, we see that this joint may fail by tearing across A, or by tearing across B and at the same time shearing the two half rivets in row A; or by tearing across C and at the same time shearing the two rivets in row B and the two half rivets in row A; therefore,

$$\text{Efficiency at } A=\frac{p-d}{p}=\cdot868.$$

We have seen that the efficiency of five rivets is ·833, so that the efficiency of one rivet

$$=\frac{\cdot833}{5}=\cdot1666.$$

$$\text{Again, efficiency at } B=\frac{p-2d}{p}+1\text{ rivet}$$

$$=\frac{10-2\cdot625}{10}+\cdot1666=\cdot7375+\cdot1666=\underline{\cdot9041.}$$

$$\text{Efficiency at } C=\frac{p-2d}{p}+3\times\cdot1666=\cdot7375+\cdot4998=\underline{1\cdot2373.}$$

The weakest part of the joint is therefore the riveting section, the efficiency of which is ·833; consequently, we must take this as the efficiency of the joint. Efficiency is then 83·3 per cent.

The Board of Trade rule for the thickness of the butt straps is

$$\frac{5}{8}\frac{\text{T}(p-d)}{(p-2d)}=\frac{5}{8}\times\frac{5}{4}\times\frac{8\cdot6875}{7\cdot373}=0\cdot92\text{ inch.}$$

The butt straps would in all probability be made 1 inch thick.

Distance between rows of rivets, etc.—

$$E=\frac{3\times d}{2}=\frac{3\times 1\frac{5}{16}}{2}=1\tfrac{31}{32}\text{ inches.}$$

$$V=\sqrt{(\tfrac{11}{20}p+d)(\tfrac{1}{20}p+d)}.$$

$$=\sqrt{\left(\frac{11\times 10}{20}+1\tfrac{5}{16}\right)\left(\frac{1\times 10}{20}+1\tfrac{5}{16}\right)}=\frac{\sqrt{3161}}{16}=3{\cdot}5\text{ inches.}$$

$$V_1=\frac{\sqrt{(11p+8d)(p+8d)}}{20}=\frac{\sqrt{2470{\cdot}25}}{20}=\frac{49{\cdot}7}{20}=2{\cdot}485=2\tfrac{1}{2}\text{ inches nearly.}$$

The joint is shown in Fig. 55.

104.—(3) The main stays in a marine boiler are pitched 16 inches apart. They are made of steel, and fitted with nuts and washers of the approved size at each side of the plate. If the boiler pressure

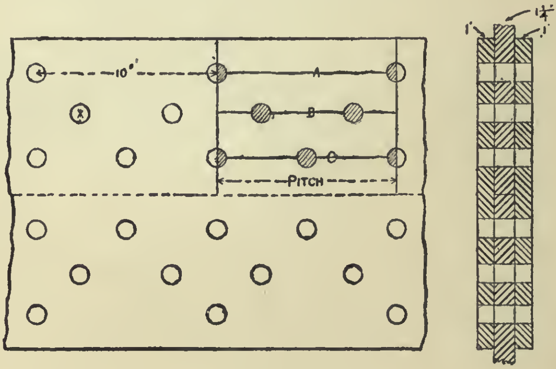

Fig. 55

is to be 180 lbs. per square inch, find the diameter of the stays and the thickness of the end plates.

Area supported by one stay $=16\times 16$ square inches.
Pressure on this area to be sustained $=16\times 16\times 180$.

Sectional area of stay $=d^2\times\frac{\pi}{4}$.

Stress allowed on a steel stay (unwelded) $=9000$ lbs.

$$\therefore\ d^2\times\frac{\pi}{4}\times 9000=16\times 16\times 180.$$

$$d^2=\frac{16\times 16\times 180}{9000\times{\cdot}7854}.$$

$$\therefore\ d=\sqrt{6{\cdot}5189}=2{\cdot}55\text{ inches diameter.}$$

The Board of Trade rule for flat surfaces is

$$\frac{C(T+1)^2}{S-6}=P,$$

where T=thickness of plate, in sixteenths of an inch;
S=surface sustained by stay;
C=constant, in this case=165;
P=boiler pressure.

$$\frac{165(T+1)^2}{250}=180. \qquad (T+1)^2=\frac{180\times250}{165}. \qquad T+1=\sqrt{272\cdot72.}$$

$$T+1=16\cdot5. \quad \therefore\ T=16\cdot5-1=15\cdot5.$$

$\therefore$ Thickness of end plate is $\frac{15\cdot5}{16}$ inch, or, say, 1 inch.

(4) The width of a combustion chamber is 36 inches, and this may be taken as the length of the stiffening girder. The stays, of which there are three, are pitched 9 inches apart, and the distance between the girders is 8 inches. If the girder stay is 10 inches deep, and $\frac{3}{4}$ inch thick, find the safe working pressure. The stress in the girder is not to exceed 11,880 lbs. per square inch.

It has been shown previously that in the Board of Trade formula for the girder stays the stress is limited to 11,880 lbs. per square inch. We can, therefore, find the boiler pressure from this formula:—

$$p=\frac{C\times d^2\times T}{(W''-P)D\times L},$$

where W″=width of combustion chamber, in inches;
P=pitch of supporting stays, in inches;
D=distance between the girders from centre to centre, in inches;
L=length of girder, in feet;
d=depth of girder, in inches;
T=Thickness of girder, in inches;
N=number of supporting stays;
$C=\frac{N\times1320}{N+1}$ in this case.

$$C=\frac{3\times1320}{4}=990.$$

$$\therefore\ p=\frac{990\times10^2\times\frac{3}{4}}{(36-9)\times8\times3}=\frac{990\times100\times3}{4\times27\times8\times3}=\underline{114\cdot5\text{ lbs.}}$$

In the above question, if the boiler pressure is 110 lbs. per square inch, find the bending moment on each of the stays.

Reaction on each support is $\frac{W}{2}$.

Weight taken by each stay is $\frac{W}{3}$.

Bending moment at centre of girder

$$=\frac{W}{2}\times\frac{L}{2}-\frac{W}{3}\times\frac{L}{4}$$

$$=\frac{WL}{4}-\frac{WL}{12}=\frac{WL}{6}.$$

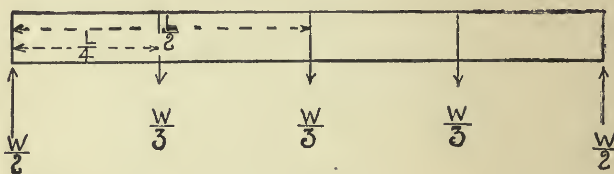

Fig. 56

The load, W, is equal to the pressure multiplied by the area upon which it acts $=(W''-P)D\times p$.

Substituting this value for W, we have,

Bending moment at centre

$$=\frac{(W''-P)D\times p\times L}{6}$$

$$=\frac{(36-9)8\times 110\times 3}{6}=\frac{27\times 8\times 110\times 3}{6}$$

$=$ 11880 pounds-feet at centre.

Bending moment on each side stay

$$=\frac{W}{2}\times\frac{L}{4}=\frac{WL}{8}.$$

Proceeding as before, giving W its value, we have

$$\frac{(36-9)8\times 110\times 3}{8}=\frac{27\times 8\times 110\times 3}{8}=8910 \text{ pounds-feet at each side.}$$

Example.—A combustion chamber is 36 inches wide, and the horizontal pitch of the tubes is $4\frac{1}{2}$ inches. If the tubes are 3 inches diameter, and the thickness of tube plate $\frac{1}{2}$ inch, would this plate pass the Board of Trade surveyor for 180 lbs. pressure?

The Board of Trade formula is $\frac{(D-d)T}{W\times D}\times 28{,}000=p$.

We have already seen that this may be written

$$\frac{2(D-d)T\times f_c}{W\times D}=p,$$

where D=horizontal pitch of tubes;
d=inside diameter of tubes;
T=thickness of tube plate;
W=width of combustion chamber, in inches;
f_c=allowable compressive stress.

Then

$$\frac{2(4\tfrac{1}{2}-3)\times\tfrac{1}{2}}{36\times4\tfrac{1}{2}}\times f_c=180.$$

$$\frac{1\cdot5f_c}{162}=180.$$

$$\therefore\ 1\cdot5f_c=180\times162.$$

$$f_c=\frac{180\times162}{1\cdot5}=\underline{19{,}440\text{ lbs.}}$$

As the compressive stress allowed is only 14,000 lbs. per square inch, the plate would not pass the Board of Trade for this pressure.

Chapter VII

SIMPLE MACHINE DESIGNS

SUSPENSION LINKS, COTTERED JOINTS, AND FOUNDATION BOLTS

105.—THE riveted joint, considered fully in the previous chapter, constitutes perhaps the simplest, but not the least important, example of machine design, the plates being exposed to tensile stress and the rivets to shear stress only. A few other examples occur in practice of parts exposed merely to simple tensile and shear stresses, the principal of which are the suspension link, the cottered joint, and the cotter foundation bolt.

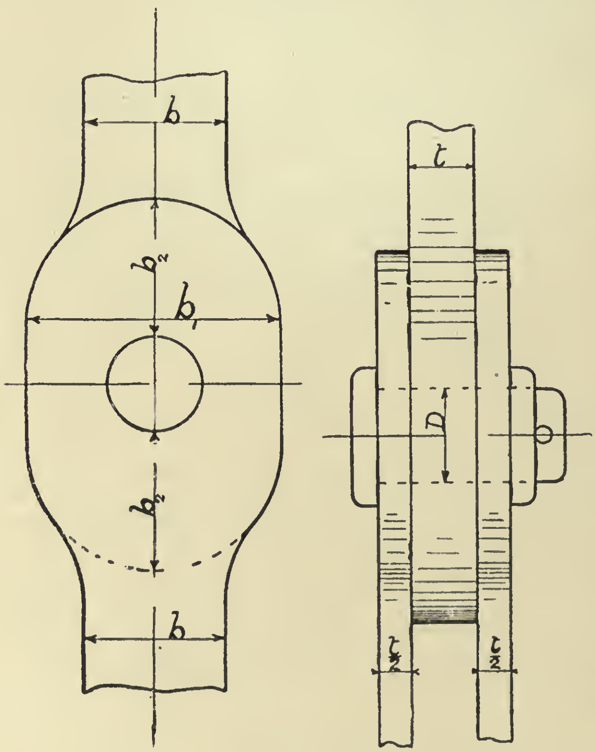

Fig. 57

The calculations of the strength of these parts are very similar to those for riveted joints, but they are not quite so simple as in the latter case, where we are able to simplify matters by fixing the diameter of the rivets to commence with. In the examples now to be considered, the diameter of the pin or bolt has to be determined by equating one resistance to another, after which other resistances have to be equated until the various dimensions of the design are satisfactorily arrived at.

106.—**Suspension Link.** The first example we shall deal with is much used in bridge work, viz. the *suspension link*, a sketch of which is given in Fig. 57. This consists simply of a flattened bar placed between two other flat bars of similar outline, and a round pin which passes through all three bars and holds them together. The pin itself is secured in position by collars, one fast and one loose, the latter being secured by a split pin.

The link is subjected to a tensile load, in consequence of which, it may fail in any of the following ways: (1) by tearing through the solid bar, either through the single inner plate or through the two outer plates; (2) by tearing the enlarged part of the inner bar, or of the two outer bars through the pin hole; (3) by shearing through the pin; (4) by crushing the pin (or the metal bearing against the pin); (5) by breaking through the part directly above (or below), the pin (depending on whether the single inner bar or the two outer bars fail), in the direction of the vertical centre line; (6) by shearing out the metal above (or below) the pin.

As regards the fifth and sixth methods of failure, the distance between the hole and the edge of the rounded outline of the bar, measured along the vertical centre line (b_2 on the drawing), is usually made such that failure by these two methods is out of the question, viz. equal to the diameter of the pin at least, so it is only necessary to consider the first four methods of failure.

Let D = the diameter of the pin, in inches.
,, b = breadth of solid part of bars, in inches.
,, t = thickness of inner bar, in inches.
,, b_1 = breadth of enlarged part of bars, in inches.
,, f_t^1 = ultimate tensile stress of bar material.
,, f_s^1 = ,, shearing ,, pin ,,
,, f_c^1 = ,, crushing ,, ,, ,,

We may assume the bars and the pin to be made of the same material, so that it matters not whether we consider crushing of the pin or of the bars. If the materials be not the same, we must consider crushing of the part made of the weaker material. The thickness of the outer bars will clearly be equal to $\frac{t}{2}$, as these are of the same width as the inner bar, and together are subject to the same load.

The tearing resistance of the inner bar at the solid part is btf_t^1. The shearing resistance of the pin, which is in double shear, is $\frac{\pi}{4}D^2f_s^1 2 = \frac{\pi}{2}D^2f_s^1$, providing the pin be a perfect fit. The crushing resistance of the pin is Dtf_c^1.

As regards the shearing resistance of the pin, it is important to notice that in actual practice we cannot assume the pin to be a perfect fit, as there must be a certain amount of clearance. If much wear take place, the amount of clearance will be considerable. The result then is that the pin, in addition to being exposed to a shearing action, may also be exposed to a slight amount of bending, which will give rise to additional stress. This bending action, however, is not important so far as the stress to which it gives rise is concerned, but it is important on account of the fact that when it occurs, its effect is to alter the distribution of the shear stress over the sections of the pin we are concerned with, causing the stress at the centre to be greater than the mean stress acting over the sections. In the case of a round section, it can be shown that the maximum stress is $\frac{4}{3}$ times the mean stress. Unless, therefore, the pin be a perfect fit, we cannot say that its shearing resistance is $\frac{\pi}{2}D^2f_s^1$. If there be clearance, and consequently bending, the true shearing resistance will be $\frac{\pi}{2}D^2f_s^1$ multiplied by the reciprocal of the ratio of maximum to mean stress, i.e. the reciprocal of $\frac{4}{3}$, which is of course $\frac{3}{4}$. The actual shearing resistance for the case in point will be then $\frac{\pi}{2}D^2f_s^1 \times \frac{3}{4} = \frac{3\pi}{8}D^2f_s^1$.

It may occur to the student that if it be necessary to make this correction for a pin joint, why did we not make a similar correction when dealing with riveted joints, because the rivets of, say, a butt joint with double cover straps, are exposed to precisely the same conditions as the pin of Fig. 57.

The answer is that in the case of the rivets, there is not, or should not be at least, any clearance whatever; each rivet is fitted into the hole whilst hot, and the head is then formed either by hammering the projecting end or squeezing it by hydraulic pressure, the result being that the metal is pressed out and against the circumference of the hole, so that there cannot be any clearance. Each rivet is thus subject to shear without bending.

107.—In order that the link joint under consideration shall be designed to the best advantage, we must so proportion the parts that the various resistances will, as far as practicable, be equal to one another.

We may determine first the diameter D of the pin.

Let F = the total tensile force acting on the link,
,, f_s = the allowable shearing stress on the pin.

Then,
$$f_s=\frac{F}{2\times\frac{3}{4}\times\frac{\pi}{4}\times D^2}=\frac{F}{\frac{3\pi}{8}D^2}=\frac{8F}{3\pi D^2}.$$

$$\therefore\ D^2=\frac{8F}{3\pi f_s},$$

from which,
$$D=\sqrt{\frac{8F}{3\pi f_s}}.$$

Having fixed the diameter of the pin, the breadth, b, of the solid part of the inner bar may next be determined.

We have already seen that,

Tearing resistance of inner bar at solid part $=btf_t^1$.

Shearing resistance of pin $=\frac{3\pi}{8}D^2f_s^1$.

Crushing resistance of pin $=Dtf_c^1$.

On looking at these expressions, the student will observe that the required breadth may be determined by equating the tearing resistance of the solid part of the bar to the crushing resistance of the pin. Note that we cannot obtain the value of b in terms of D by equating the shearing resistance of the pin either to the tearing resistance of the inner bar at the solid part or to the crushing resistance of the pin. Hence the reason for equating the tearing and the crushing resistances referred to.

Equating,
$$btf_t^1=Dtf_c^1.$$

from which,
$$b=\frac{Dtf_c^1}{tf_t^1}=\frac{Df_c^1}{f_t^1}.$$

The ratio of the ultimate crushing stress of the material to the ultimate tensile stress may be taken to be 1·6 to 1. Hence,

$$b=\frac{Df_c^1}{f_t^1}=\underline{1\cdot 6D.}$$

The next thing is to fix the thickness, t, of the inner bar, in terms of D. To do this, equate the crushing resistance of the pin to the shearing resistance. Thus,

$$Dtf_c^1=\frac{3\pi}{8}D^2f_s^1.$$

Dividing through by Df_c^1,

$$t=\frac{3\pi D^2f_s^1}{8Df_c^1}=\frac{3\pi Df_s^1}{8f_c^1}.$$

The shearing strength of the material may be taken to be equal to one-half the crushing strength, so that,

$$t=\frac{3\pi D}{8\times 2}=\underline{\cdot 589D.}$$

The same result may be obtained by substituting the value of b, viz. 1·6D, in the expression for the tearing resistance, and then equating the latter to the shearing resistance. Thus.

Tearing resistance $=btf_t^1=(1\cdot 6D)tf_t^1$.

Shearing resistance$=\frac{3\pi}{8}D^2f_s^1$.

Equating, $1\cdot 6Dtf_t^1=\frac{3\pi}{8}D^2f_s^1$,

$$t=\frac{3\pi D^2f_s^1}{8\times 1\cdot 6Df_t^1}.$$

Taking the ratio of f_s^1 to f_t^1 to be ·8, we get, finally,

$$t=\frac{3\pi D^2\times\cdot 8}{12\cdot 8D}=\underline{\cdot 589D.}$$

We have only now to determine the breadth of the two strips of metal, one on each side of the pin, measured along the horizontal centre line. Clearly, if the strength of this part of the link is to be equal to the strength of the solid part, the combined breadth should be equal to b, so that each strip will be equal to $\frac{1}{2}b$. It has been shown that $b=1\cdot 6D$, and the breadth of each strip will consequently be ·8D. The total breadth, b, of the enlarged part, will thus be $\cdot 8D+D+\cdot 8D=2\cdot 6D$.

We obtain the same result by equating the tearing resistance of the enlarged part to that of the solid part. Thus,

$$(b_1-D)tf_t^1=btf_t^1,$$

from which, $b_1-D=b$, and $b_1=b+D$.

Substituting the value of b in terms of D,

$$b_1=1\cdot 6D+D=\underline{2\cdot 6D.}$$

The proportions of the joint in terms of the diameter of the pin may be summed up as follow:—

$$b=\underline{1\cdot 6D.}$$
$$t=\underline{\cdot 589D.}$$
$$b_1=\underline{2\cdot 6D.}$$
$$b_2=\underline{D}\text{ (minimum).}$$

The method of utilising these results in the actual design of a suspension link is illustrated clearly by a worked example at the end of the chapter. It is to be understood that the proportions given only apply when the ratio of f_s^1 to f_t^1

to f_c^1 is that which has been assumed, viz. ·8 : 1 : 1·6. If this ratio be modified, the proportions will also be modified slightly.

108.—**Cottered Joint.** A joint of this type is illustrated in Fig. 58. It is employed in cases where it is desired to connect two rods securely together so that force may be transmitted in the direction of their lengths. We shall suppose the cotter to be made of the same material as the rods, generally mild steel.

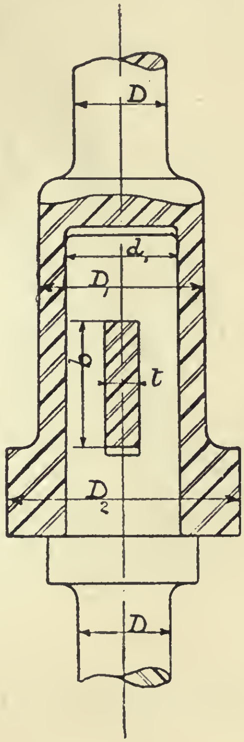

Fig. 58

The joint is exposed to the action of a tensile load, and may fail in the following ways: (1) by tearing the solid rod; (2) by tearing that part of the rod through which the cotter passes; (3) by tearing through the socket at the cotter hole; (4) by shearing through the cotter; (5) by crushing the cotter at the part which bears against the rod; (6) by crushing the cotter at the parts which bear against the socket; (7) by shearing out the metal (in the rod) which bears against the cotter; (8) by shearing out the metal (in the socket) which bears against the cotter.

The seventh and eighth methods of failure need not be considered, because in actual practice a margin of strength is provided at the parts concerned to resist failure by these two methods.

Let D = the diameter of the solid rod.
,, D_1 = the outside diameter of the small part of the socket.
,, D_2 = the outside diameter of the enlarged part of the socket.
,, d_1 = the diameter of the rod in the socket.
,, t = the thickness of the cotter.
,, b = the breadth of the cotter.

The tearing resistance for (1) is $\frac{\pi}{4}D^2 f_t^1$.

,, ,, ,, ,, (2) ,, $\left(\frac{\pi}{4}d_1^2 - d_1 t\right) f_t^1$, very nearly.

,, ,, ,, ,, (3) ,, $\left\{\frac{\pi}{4}(D_1^2 - d_1^2) - (D_1 - d_1)t\right\} f_t^1$.

,, shearing ,, ,, (4) ,, $2btf_s^1$.
,, crushing ,, ,, (5) ,, $d_1 t f_c^1$ very nearly.
,, ,, ,, ,, (6) ,, $(D_2 - d_1)tf_c^1$.

The first thing to do is to fix the diameter, D, of the solid rod. If F be the tensile force acting along the rod, and f_t the allowable tensile stress in the rod, then

$$f_t = \frac{F}{\frac{\pi}{4}D^2}.$$

Rearranging, $$\frac{\pi}{4}D^2 f_t = F, \text{ and } D^2 = \frac{4F}{\pi f_t}.$$

$$\therefore\ D = \sqrt{\frac{4F}{\pi f_t}} = 2\sqrt{\frac{F}{\pi f_t}}.$$

Having fixed the diameter D, we can next determine the value of d_1 in terms of D. To do this, equate (5) and (1).

$$d_1 t f_c^1 = \frac{\pi}{4}D^2 f_t^1.$$

Then $$d_1 t = \frac{\pi D^2 f_t^1}{4 f_c^1} = \frac{\pi D^2}{4 \times 1{\cdot}6} = {\cdot}491D^2.$$

(We are assuming f_c^1 to be equal to $1{\cdot}6\, f_t^1$.)

Now equate (2) and (1), and in (2) substitute the value of $d_1 t$ in terms of D, as just found.

$$\left(\frac{\pi}{4}d_1^2 - d_1 t\right) f_t^1 = \frac{\pi}{4}D^2 f_t^1.$$

Substituting $\cdot491D^2$ for $d_1 t$,

$$\left(\frac{\pi}{4}d_1^2 - {\cdot}491D^2\right) f_t^1 = \frac{\pi}{4}D^2 f_t^1$$

Cancelling out f_t^1,

$$\frac{\pi}{4}d_1^2 = \frac{\pi}{4}D^2 + {\cdot}491D^2 = 1{\cdot}276D^2.$$

$$d_1^2 = \frac{1{\cdot}276D^2 \times 4}{\pi} = 1{\cdot}624D^2.$$

$$\therefore\ d_1 = \sqrt{1{\cdot}624D^2} = \underline{1{\cdot}273D.}$$

Next find the value of t in terms of D by equating (5) and (1).

$$d_1 t f_c^1 = \frac{\pi}{4}D^2 f_t^1.$$

Substitute for d_1 its value in terms of D, viz. $1{\cdot}273D$.

$$1{\cdot}273D t f_c^1 = \frac{\pi}{4}D^2 f_t^1.$$

Then $$t = \frac{\pi D^2 f_t^1}{4 \times 1{\cdot}273 D f_c^1} = \frac{\pi D}{4 \times 1{\cdot}273 \times 1{\cdot}6} = \underline{{\cdot}385D.}$$

$$\therefore\ t = \underline{{\cdot}385D.}$$

The value of t may of course also be found from the equation,

$$d_1 t = \cdot 491 D^2,$$

the value of d_1 in terms of D having been determined.

Now find the value of b in terms of D by equating (4) and (1).

$$2btf_s^1 = \frac{\pi}{4} D^2 f_t^1.$$

Substitute for t its value in terms of D, viz. ·385D.

$$2b \times \cdot 385 D f_s^1 = \frac{\pi}{4} D^2 f_t^1,$$

$$b = \frac{\pi D^2 f_t^1}{4 \times 2 \times \cdot 385 D f_s^1} = \frac{\pi D}{4 \times \cdot 77 \times \cdot 8} = \underline{1 \cdot 275 D}.$$

(We are assuming f_s^1 to be equal to $\cdot 8 f_t^1$.)

$$\therefore\ b = \underline{1 \cdot 275 D}.$$

The value of D_1 can be found by equating (3) and (1).

$$\left\{\frac{\pi}{4}(D_1^2 - d_1^2) - (D_1 - d_1)t\right\} f_t^1 = \frac{\pi}{4} D^2 f_t^1.$$

As the term f_t^1 occurs on both sides of the equation, it may be cancelled out.

Substitute for d_1 and t in the left-hand side of the equation, their values in terms of D as already found.

Then we have

$$\frac{\pi}{4}\left\{D_1^2 - (1 \cdot 273 D)^2\right\} - (D_1 - 1 \cdot 273 D) \cdot 385 D = \frac{\pi}{4} D^2.$$

$$\frac{\pi}{4}\left\{D_1^2 - 1 \cdot 624 D^2\right\} - (\cdot 385 D D_1 - \cdot 491 D^2) = \frac{\pi}{4} D^2.$$

Removing the brackets,

$$\frac{\pi}{4} D_1^2 - 1 \cdot 276 D^2 - \cdot 385 D D_1 + \cdot 491 D^2 = \frac{\pi}{4} D^2.$$

Dividing through by $\frac{\pi}{4}$ (or ·785),

$$D_1^2 - 1 \cdot 624 D^2 - \cdot 491 D D_1 + \cdot 625 D^2 = D^2.$$

$$D_1^2 - \cdot 491 D D_1 = D^2 + 1 \cdot 624 D^2 - \cdot 625 D^2 = \text{say, } 2D^2.$$

We have now to solve the quadratic equation,

$$D_1^2 - \cdot 491 D D_1 = 2D^2.$$

Completing the square,

$$D_1^2 - \cdot 491 D D_1 + \left(\frac{\cdot 491 D}{2}\right)^2 = 2D^2 + \cdot 06 D^2.$$

Extracting the square root,

$$D_1 - \cdot 245 D = \sqrt{2 \cdot 06 D^2}$$
$$D_1 - \cdot 245 D = \pm 1 \cdot 433 D$$
$$D_1 = \pm 1 \cdot 433 D + \cdot 245 D$$

Clearly, the positive sign is the one which applies; hence,

$$D_1 = 1{\cdot}433D + {\cdot}245D = \underline{1{\cdot}678D.}$$

Finally, we have to determine the value of D_2 in terms of D. This can be done by equating (6) and (1).

$$(D_2 - d_1)tf_c^1 = \frac{\pi}{4}D^2 f_t^1.$$

Substituting for d_1 and t their values in terms of D,

$$(D_2 - 1{\cdot}273D){\cdot}385Df_c^1 = \frac{\pi}{4}D^2 f_t^1,$$

$$D_2 - 1{\cdot}273D = \frac{\pi D^2 f_t^1}{4 \times {\cdot}385Df_c^1} = \frac{\pi D}{1{\cdot}54 \times 1{\cdot}6} = 1{\cdot}273D.$$

$$\therefore\ D_2 = 1{\cdot}273D + 1{\cdot}273D = \underline{2{\cdot}546D.}$$

The proportions of the joint, expressed in terms of D, the diameter of the solid rod, are therefore as follow:—

$d_1 = 1{\cdot}273D.$ $D_1 = 1{\cdot}678D.$
$t = {\cdot}385D.$ $D_2 = 2{\cdot}546D.$
$b = 1{\cdot}275D.$

The above results are approximately correct to the third place of decimals, but an actual designer would of course never work to three places of decimals in fixing the dimensions of a cotter joint; this would mean working to a thousandth of an inch, whereas results correct to $\frac{1}{32}$ or to $\frac{1}{16}$ of an inch are near enough for all practical purposes.

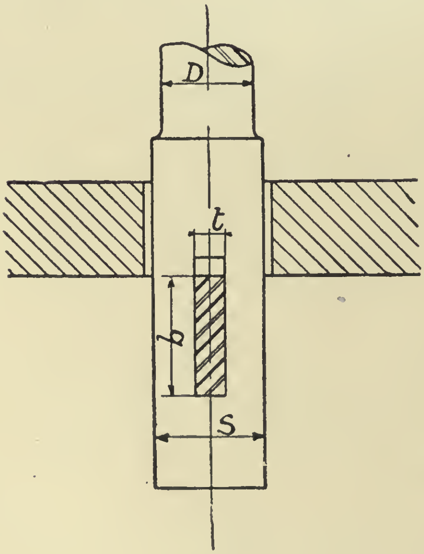

Fig. 59

109.—**Foundation Bolts.** A sketch of a foundation bolt, as used for fastening down the bed-plate of an engine to its foundation, is given in Fig. 59. The bottom portion of the bolt is enlarged and is provided with a cotter hole, through which a cotter is passed before the bolt is finally tightened up. The enlarged portion is sometimes made of round and sometimes of square section. The consideration of the strength of the bolt when the section is round is practically the same as that for the strength of the cottered joint just dealt with, and need not therefore be gone into. We will, however, consider the strength of the square section.

When the nut is tightened up, the bolt is exposed to a tensile load, which tends to cause failure in the following ways: (1) by tearing through the rounded portion of the bolt; (2) by tearing through the square section at the cotter hole; (3) by shearing the cotter; (4) by crushing the cotter.

Let D=diameter of rounded part.
,, s=the length of the side of the square section.
,, t=thickness of cotter.
,, b=breadth of cotter.

Then,

Tearing resistance for (1)$=\frac{\pi}{4}D^2 f_t^1$.
,, ,, ,, (2)$=(s^2-st)f_t^1$.
Shearing ,, ,, (3)$=2btf_s^1$.
Crushing ,, ,, (4)$=stf_c^1$.

It is of course understood that the length of the cotter is such that the combined length of the parts projecting through the bolt is at least equal to that in the bolt, as, otherwise, failure by crushing of the cotter would take place at the parts projecting and not at the part in the bolt.

We require to determine the values of s, t, and b in terms of the diameter D of the rounded part of the bolt.

First determine st in terms of D by equating (4) and (1); in order to find s in terms of D equate (2) and (1), and in (2) substitute this value of st. Thus, equating (4) and (1),

$$stf_c^1=\frac{\pi}{4}D^2 f_t^1.$$

$$st=\frac{\pi D^2 f_t^1}{4f_c^1}=\frac{\pi D^2}{4\times 1{\cdot}6}=\underline{{\cdot}491D^2}.$$

Equating (2) and (1), and substituting $\cdot491D^2$ for st in (2),

$$(s^2-st)f_t^1=\frac{\pi}{4}D^2 f_t^1.$$

$$(s^2-{\cdot}491D^2)f_t^1=\frac{\pi}{4}D^2 f_t^1.$$

The term f_t^1 cancels out. Then,

$$s^2-{\cdot}491D^2=\frac{\pi}{4}D^2.$$

$$s^2={\cdot}785D^2+{\cdot}491D^2=1{\cdot}276D^2.$$

$$s=\sqrt{1{\cdot}276D^2}=\underline{1{\cdot}129D}.$$

To find t, equate (4) and (1), and substitute 1·129D for s in (4).

$$stf_c^1 = \frac{\pi}{4}D^2 f_t^1.$$

$$1{\cdot}129Dtf_c^1 = \frac{\pi}{4}D^2 f_t^1.$$

$$t = \frac{\pi D^2 f_t^1}{4 \times 1{\cdot}129 D f_c^1} = \frac{\pi D}{4 \times 1{\cdot}129 \times 1{\cdot}6} = \underline{{\cdot}435D.}$$

$$\therefore t = \underline{{\cdot}435D}.$$

Now find b by equating (3) and (1), and substituting ·435D for t in (3).

$$2btf_s^1 = \frac{\pi}{4}D^2 f_t^1,$$

$$2b \times {\cdot}435Df_s^1 = \frac{\pi}{4}D^2 f_t^1,$$

$$b = \frac{\pi D^2 f_t^1}{4 \times 2 \times {\cdot}435 D f_s^1} = \frac{\pi D}{8 \times {\cdot}435 \times {\cdot}8} = \underline{1{\cdot}129D.}$$

$$\therefore b = \underline{1{\cdot}129D}.$$

The proportions for the cotter bolt, when the enlarged part is of square section, are consequently,

$$s = 1{\cdot}129D,$$
$$t = {\cdot}435D,$$
$$b = 1{\cdot}129D.$$

It will be noted that the length of the side of the square section is equal to the breadth of the cotter.

It may be well to remind the student that the results deduced in the foregoing are based on purely theoretical considerations, and in many actual examples of suspension links, etc., the proportions often differ considerably from those given, practical experience perhaps having shown that a certain dimension might with advantage be reduced somewhat, whilst another might be increased, and so on.

Worked Examples

110.—(1) Determine the principal dimensions of a mild steel suspension link which is required to support a load of 35 tons.

Ultimate tensile stress of steel, 30 tons per square inch.
,, shear ,, ,, 24 ,, ,,
,, crushing ,, ,, 50 ,, ,,

In this question, it must be noted that we cannot, strictly speaking, adopt the proportions determined in the text, because the relative values of the tensile, shear, and crushing strengths of the material are somewhat different from those assumed in the text. The difference is, however, only slight, and we might adopt the proportions referred to without appreciable error, but it will be instructive to work the problem from first principles.

The diameter of the pin must first be determined.

Let D = the diameter of the pin, in inches.
,, f_s = the allowable shearing stress of the steel.
,, F = the load on the link = 35 tons.

It has been shown that,

$$D=\sqrt{\frac{8F}{3\pi f_s}}.$$

In the question, we are given not the allowable but the ultimate shear stress of the steel. We must therefore divide the ultimate stress by a suitable factor of safety to decide the safe stress to be adopted. For a suspension link, a factor of safety of 6 may be adopted. The allowable stress is then $24 \div 6 = 4$ tons per square inch.

Substituting the known quantities in the above expression,

$$D=\sqrt{\frac{8\times 35}{3\times\pi\times 4}}=\sqrt{7{\cdot}425}=\underline{2{\cdot}72\text{ inches.}}$$

Required diameter of pin = 2·72 inches.

Let b = breadth of solid part of bars, in inches.
,, t = thickness of inner bar, in inches.
,, b_1 = breadth of enlarged part of bars, in inches.

To determine b, equate the tearing resistance of the solid part of the inner bar to the crushing resistance of the pin.

$$b\times t\times 30=2{\cdot}72\times t\times 50.$$

$$b=\frac{2{\cdot}72\times t\times 50}{t\times 30}=\underline{4{\cdot}53\text{ inches.}}$$

Required breadth of solid part of bars = 4·53 inches.

To determine t, equate the crushing resistance of the pin to the shearing resistance.

$$2{\cdot}72\times t\times 50=\frac{\pi}{4}\times 2{\cdot}72^2\times 24\times\frac{3}{4}\times 2.$$

(Note that the factor, $\frac{3}{4}$, is introduced to make allowance for bending, as explained in the text.)

$$t=\frac{\pi\times 2{\cdot}72^2\times 24\times 3\times 2}{4\times 2{\cdot}72\times 50\times 4}=\underline{1{\cdot}536\text{ inches.}}$$

Required thickness of inner bar = 1·536 inches.

The combined thickness of the outer bars must be equal to the thickness of the inner bar, the breadth being the same. Hence,

Thickness of outer bars = $\frac{1}{2}$ of 1·536 = ·768 inch.

The amount of metal resisting tearing at the pin hole in the enlarged part of the bars must be equal to the amount of metal in the solid bars. Since the thickness is the same as that of the corresponding solid bars, the combined breadth of the two strips, one on each side of the pin, measured along the horizontal centre line, will clearly be equal to b, or 4·53 inches. The breadth, b_1, of the enlarged part will consequently be equal to

$$b + D = 4{\cdot}53 + 2{\cdot}72 = \underline{7{\cdot}25 \text{ inches.}}$$

Breadth of enlarged part of bars measured along the horizontal centre through the pin hole $= \underline{7{\cdot}25 \text{ inches.}}$

The distance from the circumference of the pin hole to the edge of the link, measured along the vertical centre line, may be made equal to the diameter of the pin.

Summing up, the required dimensions are as follow :—

Diameter of pin	=2·72 inches, or, say,	$2\frac{3}{4}$ inches.
Breadth of solid bar	=4·53 ,, ,,	$4\frac{9}{16}$,,
Thickness of inner bar	=1·536 ,, ,,	$1\frac{9}{16}$,,
Thickness of outer bars	= ·768 inch, ,,	$\frac{13}{16}$ inch.
Breadth of enlarged part	=7·25 inches, ,,	$7\frac{5}{16}$ inches.

(2) Design a suspension link to carry a load of 25 tons, the safe shear stress on the pin being taken at 4·5 tons per square inch. Assume the ultimate shear stress to be ·8 of the ultimate tensile and ·5 of the ultimate crushing stress of the material, the pin being of the same material as the link.

The first part of this question, which is to determine the diameter of the pin, is similar to the first part of the previous question. For the second part, we may make use of the proportions given in the text, because the ratio of the ultimate shear, tensile, and crushing stresses of the material is the same as that employed in the text to deduce the proportions.

To find the diameter of the pin, D, use the relation,

$$D = \sqrt{\frac{8F}{3\pi f_s}},$$

where F is the load on the link, and f_s the allowable shear stress on the pin.

Substituting the given values,

$$D = \sqrt{\frac{8 \times 25}{3 \times \pi \times 4{\cdot}5}} = \sqrt{4{\cdot}72} = \underline{2{\cdot}17 \text{ inches.}}$$

Diameter of pin $= \underline{2{\cdot}17 \text{ inches.}}$

We found in the text that with the ultimate stresses in the ratio given in the question, the following proportions should be adopted:—

$$b=1{\cdot}6\text{D},$$
$$t={\cdot}589\text{D},$$
$$b_1=2{\cdot}6\text{D},$$
$$b_2=\text{D},$$

where b=breadth of solid part of bars, in inches;
t=thickness of inner bar, in inches;
b_1=breadth of enlarged part of bars, in inches;
b_2=distance from circumference of pin hole to edge of bar, measured along the vertical centre line.

D we have just found to be 2·17 inches.

Hence, $b=1{\cdot}6\text{D}=1{\cdot}6\times2{\cdot}17$ inches$=3{\cdot}47$ inches;
$t={\cdot}589\text{D}={\cdot}589\times2{\cdot}17$,, $=1{\cdot}278$,,
$b_1=2{\cdot}6\text{D}=2{\cdot}6\times2{\cdot}17$,, $=5{\cdot}64$,,
$b_2=\text{D}$ $=2{\cdot}17$,, (minimum).

The required dimensions are then,

Diameter of the pin	=2·17 inches,	or, say,	$2\frac{1}{4}$ inches.
Breadth of solid bars	=3·47 ,,	,,	$3\frac{1}{2}$,,
Thickness of inner bar	=1·278 ,,	,,	$1\frac{5}{16}$,,
Thickness of outer bars	= ·639 ,,	,,	$\frac{21}{32}$ inch.
Breadth of enlarged part	=5·64 ,,	,,	$5\frac{3}{4}$ inches.

111.—(3) Two mild steel rods, $1\frac{1}{2}$ inches diameter, are to be connected together by a cottered joint of the type shown in Fig. 58. Assuming the crushing strength, the tensile strength, and the shear strength of the materials to be in the ratio, 11 : 6 : 5, design the joint. Assume the cotter to be of the same material as the rods.

Let D=the diameter of the solid rod=$1\frac{1}{2}$ inches.
,, D_1= ,, outside diameter of the small part of the socket.
,, D_2= ,, ,, ,, enlarged part of the socket.
,, d_1= ,, diameter of the rod in the socket.
,, t= ,, thickness of the cotter.
,, b= ,, breadth ,, ,,

First equate the crushing resistance of the cotter at the part which bears against the rod to the tearing resistance of the solid rod. Thus,

$$d_1tf_c^1=\frac{\pi}{4}\text{D}^2f_t^1,$$

where f_c^1 and f_t^1 represent respectively the ultimate crushing stress of the cotter material and the ultimate tensile stress of the rod material. These are in the ratio of 11 to 6.

$$d_1t=\frac{\pi\text{D}^2f_t^1}{4f_c^1}=\frac{\pi\text{D}^2 6}{4\times11}={\cdot}428\text{D}^2.$$

Now equate the tearing resistance of that part of the rod through which the cotter passes to the tearing resistance of the solid rod.

$$\left(\frac{\pi}{4}d_1^2-d_1t\right)f_t^1=\frac{\pi}{4}D^2f_t^1.$$

Substitute for d_1t the value $\cdot428D^2$ just found, and cancel out f_t^1.

$$\frac{\pi}{4}d_1^2-\cdot428D^2=\frac{\pi}{4}D^2.$$

$$\frac{\pi}{4}d_1^2=\cdot785D^2+\cdot428D^2=1\cdot213D^2.$$

$$d_1^2=\frac{1\cdot213D^2\times4}{\pi}=1\cdot545D^2.$$

$$d_1=\sqrt{1\cdot545D^2}=1\cdot242D.$$

We are told in the question that $D=1\frac{1}{2}$ inches,

$$\therefore\ d_1=1\cdot242D=1\cdot242\times1\cdot5=\underline{1\cdot863\text{ inches.}}$$

The value of t can next be found from the equation,

$$d_1t=\cdot428D^2,$$

the value of d_1 being now known. Thus, $d_1=1\cdot242D$.

$$t=\frac{\cdot428D^2}{d_1}=\frac{\cdot428D^2}{1\cdot242D}=\cdot344D.$$

Now $D=1\cdot5$ inches. Therefore, $t=\cdot344\times1\cdot5=\underline{\cdot516\text{ inch.}}$

To find the value of b, equate the shearing resistance of the cotter to the tearing resistance of the solid rod.

$$2btf_s^1=\frac{\pi}{4}D^2f_t^1,$$

where f_s^1=the ultimate shear stress of the cotter material.

$$b=\frac{\pi D^2f_t^1}{4\times2tf_s^1}.$$

The ratio of f_t^1 to f_s^1 is given as $6:5$, and t we have found to be equal to $\cdot344D$. Substituting,

$$b=\frac{\pi D^2 6}{4\times2\times\cdot344D\times5}=1\cdot37D.$$

Now $D=1\cdot5$ inches.

$$\therefore\ b=1\cdot37\times1\cdot5=\underline{2\cdot055\text{ inches.}}$$

D_1 may next be found by equating the tearing resistance of the socket at the cotter hole to the tearing resistance of the solid rod. Thus,

$$\left\{\frac{\pi}{4}(D_1^2-d_1^2)-(D_1-d_1)t\right\}f_t^1=\frac{\pi}{4}D^2f_t^1.$$

The factor f_t^1 cancels out, of course.

We showed in the text, by solving this equation, that $D_1=1·678D$.

Hence, $D_1=1·678\times 1·5=\underline{2·517 \text{ inches.}}$

Finally, to find D_2, equate the crushing resistance of the cotter at the parts which bear against the socket to the tearing resistance of the solid rod.

$$(D_2-d_1)tf_c^1=\frac{\pi}{4}D^2f_t^1.$$

Substitute for d_1 its value in terms of D, viz. 1·242D, and for t the value ·344D.

$$(D_2-1·242D)·344Df_c^1=\frac{\pi}{4}D^2f_t^1.$$

$$D_2-1·242D=\frac{\pi D^2f_t^1}{4\times·344Df_c^1}=\frac{2·282D\times 6}{11}=\underline{1·242D}.$$

$$D_2=1·242D+1·242D=2·484D.$$

$$\therefore\ D_2=2·484\times 1·5=\underline{3·726 \text{ inches.}}$$

The dimensions of the joint are, therefore, as follow :—

$D=1·5$	inches	=say,	$1\frac{1}{2}$ inches.
$d_1=1·863$	,,	= ,,	$1\frac{7}{8}$,,
$t=·516$	,,	= ,,	$\frac{1}{2}$ inch.
$b=2·055$	,,	= ,,	$2\frac{1}{16}$ inches.
$D_1=2·517$	,,	= ,,	$2\frac{1}{2}$,,
$D_2=3·726$	,,	= ,,	$3\frac{3}{4}$,,

(4) A foundation bolt is 2 inches diameter. If the enlarged part at the bottom of the bolt is of round section, determine the diameter of this part of the bolt. Also find the breadth and thickness of the cotter. Assume the bolt to be made of wrought iron having an ultimate tensile stress of 22 tons per square inch, and the cotter of mild steel having an ultimate shear stress of 24 tons per square inch and an ultimate crushing stress of 50 tons per square inch.

In this question, the bolt is of wrought iron and the cotter of steel. The crushing strength of the wrought iron is not given. This would probably be less than that of the steel, in which case crushing of the bolt would take place before crushing of the cotter. The strictly correct way of dealing with the question would then be to consider the crushing resistance of the bolt instead of the crushing resistance of the cotter. Since, however, the crushing strength of the wrought iron is not stated, it is evidently intended that we should assume the bolt to be as strong to resist crushing as the cotter.

Let D=diameter of the body of the bolt.
,, d_1= ,, ,, ,, enlarged part.
,, t=thickness of the cotter.
,, b=breadth ,, ,,
,, f_t^1=ultimate tensile stress of the bolt material.
,, f_s^1= ,, shear stress ,, cotter ,,
,, f_c^1= ,, crushing stress ,, ,, ,,

First equate the crushing resistance of the cotter to the tearing resistance of the bolt.

$$d_1tf_c^1=\frac{\pi}{4}D^2f_t^1.$$

$$d_1t=\frac{\pi D^2 f_t^1}{4f_c^1}=\frac{\pi D^2 22}{4\times 50}=\cdot345D^2.$$

Equate next the tearing resistance of the enlarged part of the bolt to that of the bolt.

$$\left(\frac{\pi}{4}d_1^2-d_1t\right)f_t^1=\frac{\pi}{4}D^2f_t^1.$$

Substitute for d_1t the value $\cdot345D^2$, and cancel out f_t^1.

$$\frac{\pi}{4}d_1^2-\cdot345D^2=\frac{\pi}{4}D^2.$$

$$\frac{\pi}{4}d_1^2=\cdot785D^2+\cdot345D^2=1\cdot13D^2.$$

$$d_1^2=\frac{1\cdot13D^2\times4}{\pi}=1\cdot44D^2.$$

$$d_1=\sqrt{1\cdot44D^2}=1\cdot2D.$$

We are given that D=2 inches.

$$\therefore\ d_1=1\cdot2\times2=\underline{2\cdot4\text{ inches.}}$$

To find t, we may equate the crushing resistance of the cotter to the tearing resistance of the bolt, or we may use the equation,

$$d_1t=\cdot345D^2.$$

From this equation,

$$t=\frac{\cdot345D^2}{d_1}=\frac{\cdot345D^2}{1\cdot2D}=\cdot287D.$$

$$\therefore\ t=\cdot287\times2=\underline{\cdot574\text{ inch.}}$$

To find b, equate the shearing resistance of the cotter to the tearing resistance of the bolt.

$$2btf_s^1=\frac{\pi}{4}D^2f_t^1.$$

Substituting $\cdot 287D$ for t,

$$2b \times \cdot 287 D f_s^1 = \frac{\pi}{4} D^2 f_t^1.$$

$$b = \frac{\pi D^2 \times f_t^1}{4 \times 2 \times \cdot 287 D \times f_s^1}.$$

$$f_t^1 = 22 \text{ and } f_s^1 = 24.$$

Then,
$$b = \frac{\pi D \times 22}{8 \times \cdot 287 \times 24} = 1 \cdot 254 D.$$

Substituting the known value of D,

$$b = 1 \cdot 254 \times 2 = \underline{2 \cdot 508 \text{ inches.}}$$

Hence,

Diameter of enlarged portion of bolt	$= 2 \cdot 4$ inches, or, say,	$2\frac{1}{2}$ inches.
Thickness of cotter	$= \cdot 574$ inch, ,,	$\frac{5}{8}$ inch.
Breadth ,,	$= 2 \cdot 508$ inches, ,,	$2\frac{1}{2}$ inches.

Chapter VIII

STRENGTH OF SHAFTS

112.—THE object of a shaft is to transmit energy by rotation, and in order that it may do so, it is necessary to apply a turning force. The point of application of this force must clearly be at some distance from the centre or axis of the shaft.

The force required to rotate the shaft may be applied to the rim of a pulley or to the end of a lever secured to the shaft. As a result of the force applied in this manner, the shaft is said to be subject to twisting or *torsion*, and the tendency is for the shaft to fail by shearing on planes at right angles to the axis of the shaft.

113.—**Twisting Moment.** The product of the force applied and the leverage of action of the force is termed the *twisting moment* or the *turning moment*. (The term *torque* is also commonly employed to mean the same thing.)

If the force be measured in pounds and the leverage in inches, the twisting moment is said to be so many inch-pounds (or pound-inches), whilst if the force be in tons and the leverage in feet, the twisting moment is measured as so many foot-tons (or ton-feet).

The student of Mechanics knows that the terms inch-pound, foot-pound, etc., are used to represent units of work, the foot-pound, for instance, representing the work done in overcoming a resistance of one pound through a space of one foot. It is important, therefore, that the inch-pound or the foot-pound, as used in connection with the twisting moment of a shaft, should not be confused with the inch-pound or the foot-pound, as used to denote work.

In order to avoid confusion, it is convenient, in expressing twisting moments, to write *pound-inches* instead of inch-pounds, and *pound-feet* instead of foot-pounds, and in what follows, therefore, we shall express a twisting moment as so many pound-inches or pound-feet, etc. Thus, if a force of 100 lbs. be applied to the end of a lever to turn a shaft, the length of the lever being 12 inches (measured from the centre of the shaft), we say that the twisting moment is $100 \times 12 = 1200$ pound-inches, and so on.

In practical problems on the strength of shafts, where we desire to know the twisting moment, the leverage at which the driving force acts is usually known, but the actual force is often not known and has to be calculated before the twisting moment can be determined.

Take, for example, a line shaft driven by a belt lapped round a pulley keyed to the shaft. The driving force acts at the rim of the pulley, and its leverage is obviously equal to the radius of the pulley, but as the force is not known we cannot say what the twisting moment is. If, however, the speed of rotation of the shaft and the horse power the shaft transmits are known, the twisting moment may be calculated in the following manner:—

Let F = the effective driving force, in pounds.
,, L = leverage at which F acts, in inches.
,, N = number of revolutions made by shaft per minute.
,, H.P. = horse power transmitted by shaft.

The force F passes through a distance equal to the circumference of a circle whose radius is L, every revolution of the shaft, so that the work done per revolution is equal to $F \times 2\pi \times L$ inch-pounds. The work done per minute by F will be $F \times 2\pi \times L \times N$ inch-pounds. Now one horse power is equivalent to 33,000 foot-pounds per minute, or to $12 \times 33{,}000$ inch-pounds, so that the number of inch-pounds of work the shaft is transmitting per minute is $\text{H.P.} \times 12 \times 33{,}000$. This is, of course, equivalent to the work done by F in one minute.

We have then this equation,

$$F \times 2\pi \times L \times N = \text{H.P.} \times 12 \times 33{,}000.$$

The twisting moment, which may be denoted by T.M., is $F \times L$, and from the above equation,

$$F \times L = \frac{\text{H.P.} \times 12 \times 33{,}000}{2\pi \times N} = \frac{63{,}000\ \text{H.P.}}{N}.$$

Hence, $$\text{T.M.} = \frac{63{,}000\ \text{H.P.}}{N} \text{ (pound-inches).}$$

Thus, if the horse power a shaft is transmitting and the speed of the shaft are known, the twisting moment is readily obtained by multiplying the power by 63,000 and dividing by the speed of the shaft in revolutions per minute.

114.—**Strength of Shafts.** It has been pointed out that as a result of the driving force applied to rotate a shaft, there is a tendency for the shaft to fail by shearing on planes at right angles to its axis. This tendency to fail is resisted by the shearing resistance of the shaft material.

We shall now derive an equation connecting the twisting moment, the shear stress in the material, and the diameter of the shaft, which we assume to be of circular section. (It is, of course, understood that shafts are almost always made round, but in some instances, in order to meet special requirements, they are made of square section, except at the journals.)

The student who does not possess an elementary knowledge of the integral calculus may omit the following consideration, because this is only given to show how the required equation is arrived at. The main thing is, of course, the application of the equation to the solution of practical problems.

Fig. 60 is given to illustrate how a shaft is strained when subjected to a twisting moment. Suppose the shaft to be fixed at one end, A, and to be subjected to a twisting moment applied at the other end, B. Then it is clear that if, prior to the application of the twisting moment, a line had been drawn along the outer surface of the shaft, parallel to the axis, this line will be deflected as indicated in the figure, to form an angle with its original position.

Fig. 60

If now we imagine the shaft to be made up of a large number of thin discs, it will be understood that each disc, neglecting the one at the extreme left end, will be rotated slightly, the amount of the rotation being greater the further the disc is from the left end. An originally vertical diameter on any disc except the one at the left end, which is fixed, would therefore be thrown out of the vertical, as indicated in Fig. 61, after the twisting moment had been applied.

The strain at the outer circumference of the disc is represented by the arc mn, and the length of the arc between the original and the new diameter at any particular radius will represent the strain in the disc at that radius. Clearly, the strain at any radius is proportional to the radius, and as the strain is proportional to the stress which produces it, it follows that the shear stress in the disc or the shaft at any point is proportional to the distance of that point from the centre of the shaft.

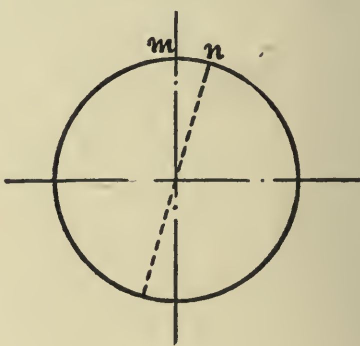

Fig. 61

Hence the stress varies uniformly from nothing at the centre of the shaft to a maximum at the outer circumference.

In considering the strength of any piece of machinery, it is the maximum stress which must be taken account of, and so in the case of a shaft, it is the stress at the external surface of the shaft which concerns us

115.—We shall deal first with a solid shaft.

Let D=the diameter of the shaft, in inches.
„ R=the radius „ „ „
„ f_s=the shear stress at the outer circumference of the shaft, in lbs. per square inch.
„ T.M.=the twisting moment, in pound-inches.

Consider any ring of the shaft (see Fig. 62), of width δx, at a distance x from the centre of the shaft. Any such ring as this is exposed to shear stress. Now in the case of simple shear, the stress is constant over the whole area of the section concerned, but in the present case it varies, as we have seen, from nothing at the centre to a maximum at the outer circumference of the shaft, being proportional to the distance from the centre.

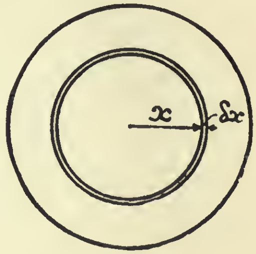

Fig. 62

The stress on the small ring under consideration will consequently be

$$f_s \times \frac{x}{R} = \frac{x}{\frac{1}{2}D} f_s = \frac{2x}{D} f_s.$$

The total stress over the ring will be the area of the ring multiplied by the stress, and this is

$$2\pi x \delta x \times \frac{2x}{D} f_s = \frac{4\pi f_s x^2}{D} \delta x.$$

The last expression represents the resistance offered by the ring to shearing, and this resistance is in the nature of a moment. The moment, taken about the centre of the shaft, is

$$\frac{4\pi f_s x^2}{D} \delta x \times x = \frac{4\pi f_s x^3}{D} \delta x.$$

Assuming now δx to become smaller and smaller, and summing up all such moments acting over the section of the shaft between the limits $x=0$ and $x=\frac{1}{2}D$,

$$\text{Total moment of resistance} = \frac{4\pi f_s}{D} \int_0^{\frac{1}{2}D} x^3 dx$$

$$= \frac{4\pi f_s}{D} \left[\frac{x^4}{4} \right]_0^{\frac{1}{2}D}$$

$$= \frac{4\pi}{D} f_s \left\{ \frac{(\frac{1}{2}D)^4}{4} - 0 \right\}$$

$$= \frac{4\pi}{D} f_s \frac{D^4}{64} = \frac{\pi}{16} D^3 f_s.$$

The expression $\frac{\pi}{16}D^3f_s$ represents the total amount of resistance of the whole section of the shaft, and this is generally known as the *torsional resistance* of the shaft. The torsional resistance must obviously balance the twisting moment, and so we have, finally,

$$T.M.=\frac{\pi}{16}D^3f_s.$$

This is the general formula for the strength of a solid round shaft, and it is very frequently required by the machine designer and the engineer, as the diameter of a shaft to resist a given twisting moment can readily be obtained from it.

It will be observed, on referring to the formula, that *the strength of a solid round shaft to resist a twisting moment varies as the cube of its diameter.* That is to say, if we double the diameter of a shaft, we make it eight times as strong as it was originally. Thus, if the diameters of three solid shafts made of the same material are in the ratio of 1 : 2 : 3, their relative strengths will be as $1^3 : 2^3 : 3^3$, or as 1 : 8 : 27, so that a 4-inch shaft is eight times as strong as a 2-inch shaft, whilst a 6-inch shaft is twenty-seven times as strong as the 2-inch and $\frac{27}{8}$ times as strong as the 4-inch shaft.

116.—It is sometimes useful to remember that a good mild steel shaft, 1 inch diameter, is capable of sustaining a maximum twisting moment of approximately 10,500 pound-inches. That this is so may be easily proved by the formula,

$$T.M.=\frac{\pi}{16}D^3f_s.$$

The shearing strength of mild steel may be taken to be 24 tons per square inch. For a 1-inch shaft, D=1, so that,

$$T.M.=\frac{\pi}{16}\times1^3\times24\times2240=10{,}550 \text{ pound-inches.}$$

Bearing the figure 10,500 in mind, we can easily arrive at the maximum twisting moment which may be applied to mild steel shafts of any diameter, because we now know that the strength of a shaft varies as the cube of its diameter. Take, for example, a $1\frac{3}{4}$-inch mild steel shaft. The relative strengths of a 1-inch and a $1\frac{3}{4}$-inch shaft will be as $1^3 : 1{\cdot}75^3$ or as 1 : 5·35. If then the maximum twisting moment which may be applied to a 1-inch shaft is 10,500 pound-inches, the maximum for a $1\frac{3}{4}$-inch shaft will be $10{,}500\times5{\cdot}35=56{,}175$ pound-inches. This means to say that a maximum force of 561·75 pounds at a leverage of 100 inches, or 5617·5 pounds at a leverage of 10 inches, and so on, might be applied to the shaft before causing failure.

For a 1-inch wrought-iron shaft, the maximum twisting moment which can be applied may be taken as 9600 pound-inches.

117.—**Polar Moment of Inertia**. The product of any mass or area, and the square of its perpendicular distance from any point, may be defined as the moment of inertia of that mass or area about that point.

In the case of a shaft, we can deduce an expression for its moment of inertia about the axis of rotation in much the same way as we deduced the expression for its moment of resistance.

Referring to Fig. 62, consider again the ring of the shaft of width δx, its distance from the centre being x.

The area of the small ring is $2\pi x\delta x$, and the polar moment of inertia of this area about the centre is $2\pi x\delta x \times x^2$. Assuming now δx to become smaller and smaller, the summing up of all such expressions as this over the whole section of the shaft between the limits $x=0$ and $x=\frac{D}{2}$ gives us the moment of inertia of the section. Thus,

$$\text{Moment of inertia}=2\pi\int_0^{\frac{1}{2}D} x^3dx$$

$$=2\pi\left[\frac{x^4}{4}\right]_0^{\frac{1}{2}D}$$

$$=2\pi\frac{\left(\frac{D}{2}\right)^4}{4}$$

$$=\frac{2\pi D^4}{4\times 16}.$$

$$\therefore\ \text{Moment of inertia}=\frac{\pi D^4}{32}\ \text{for a solid shaft.}$$

For a hollow shaft proceed the same as before, but integrate between the limits $x=\frac{d}{2}$ and $x=\frac{D}{2}$. Then,

$$\text{Moment of inertia}=2\pi\int_{\frac{1}{2}d}^{\frac{1}{2}D} x^3dx$$

$$=2\pi\left[\frac{x^4}{4}\right]_{\frac{1}{2}d}^{\frac{1}{2}D}$$

$$=\frac{2\pi}{4}\left[\frac{D^4}{16}-\frac{d^4}{16}\right]$$

$$\text{Moment of inertia}=\frac{\pi}{32}[D^4-d^4]\ \text{for a hollow shaft.}$$

118.—**The Power which may be safely Transmitted and the Speed of Shafts.** Power is the rate of doing work. A machine which does a certain amount of work in a given time when running at a certain speed will do double the work in the same time if its speed be doubled, so that by doubling the speed of a machine, the power is also doubled.

Similarly, the power which may be transmitted by a shaft is directly proportional to the speed at which the shaft runs. A shaft transmitting 100 H.P. at a speed of 100 revolutions per minute will transmit 200 H.P. at a speed of 200 revolutions per minute. Now the question we really desire to call attention to is this:—If the power transmitted by a shaft is to be increased by a corresponding increase in the speed, is it necessary to increase also the diameter of the shaft? It seems natural to suppose that a shaft, designed originally to transmit a certain power when running at a certain speed, would not be strong enough to transmit say double the original power. As a matter of fact, however, this is not the case if the speed of the shaft be doubled. The real factor to be considered from the strength point of view is the twisting moment. Providing this remain unaltered, the shaft will be quite capable of transmitting a greater amount of power than what it was originally intended to. *If the speed of rotation be increased in proportion to the increased power to be transmitted, the twisting moment is not affected in any way, and consequently the shaft is able to transmit safely the increased power.*

If, however, a greater amount of power is to be transmitted at the same speed, the twisting moment will be increased, and as a result the shaft will not now be sufficiently strong for its heavier duty, unless, of course, there be a large margin of strength.

That the twisting moment is not altered when the speed of a shaft is increased proportionally to the horse power may be readily seen by referring to the equation connecting the twisting moment with the horse power and the speed. The equation referred to is,

$$\text{T.M.} = \frac{63{,}000 \text{ H.P.}}{\text{N}} \text{ (pound-inches).}$$

Clearly, if both the H.P. and N be doubled, T.M. remains the same. If, however, H.P. be increased whilst N remains unaltered, T.M. is increased.

Another useful thing to remember in connection with the power which may be transmitted by shafts is that, for any particular speed, *the power which may be transmitted varies as the cube of the diameter.* Thus, a 2-inch shaft will transmit eight times as much power as a 1-inch shaft running at the same speed. That this is so may be seen by equating the twisting moment, expressed in terms of the power and the speed, to the torsional resistance of the shaft. Thus,

$$\frac{63{,}000\ \text{H.P.}}{\text{N}}=\frac{\pi}{16}\text{D}^3 f_s.$$

Then,
$$\text{H.P.}=\frac{\pi\text{D}^3 f_s\text{N}}{16\times 63{,}000}.$$

119.—**Strength of Hollow Round Shaft.** Large shafts, such as engine crank shafts, are frequently made hollow, with the object of obtaining greater strength for a given weight of metal. The strength of a hollow shaft may be determined by a simple integration, as in the case of a solid shaft. In fact, the consideration of the strength of a hollow shaft is practically the same as that for a solid shaft, the only difference being that the integration is performed between different limits.

Let D = external diameter of shaft, in inches.
,, d = internal ,, ,, ,,
,, f_s = the shear stress at the outer circumference of the shaft, in lbs. per square inch.
,, T.M. = the twisting moment, in pound-inches.

Then considering again any ring of the shaft of width δx at a distance x from the centre of the shaft, we have seen that the total stress over the ring is $\frac{4\pi f_s x^2 \cdot \delta x}{\text{D}}$, and the moment of this $\frac{4\pi f_s x^3}{\text{D}} \cdot \delta x$.

For a hollow shaft the value of x varies from $\frac{1}{2}d$ to $\frac{1}{2}\text{D}$, instead of from 0 to $\frac{1}{2}\text{D}$ as in the case of the solid shaft. We must, therefore, integrate the expression $\frac{4\pi f_s x^3 \cdot \delta x}{\text{D}}$ between the limits $\frac{1}{2}d$ and $\frac{1}{2}\text{D}$. Then,

$$\begin{aligned}\text{T.M.}&=\frac{4\pi f_s}{\text{D}}\int_{\frac{1}{2}d}^{\frac{1}{2}\text{D}} x^3 dx\\ &=\frac{4\pi f_s}{\text{D}}\left[\frac{x^4}{4}\right]_{\frac{1}{2}d}^{\frac{1}{2}\text{D}}\\ &=\frac{4\pi f_s}{\text{D}}\left\{\frac{(\frac{1}{2}\text{D})^4}{4}-\frac{(\frac{1}{2}d)^4}{4}\right\}\\ &=\frac{4\pi f_s}{\text{D}}\left(\frac{\text{D}^4-d^4}{64}\right)\\ &=\frac{\pi}{16}\left(\frac{\text{D}^4-d^4}{\text{D}}\right)f_s.\end{aligned}$$

For a hollow shaft, therefore, the general equation is

$$\text{T.M.} = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right) f_s.$$

120.—Comparison of Solid and Hollow Shafts. It has been stated that, weight for weight, a hollow shaft is stronger than a solid shaft, and we shall now compare the torsional resistances of two shafts of equal weight and length and made of the same material, one solid and the other hollow.

In deriving the formula for the twisting moment for both the solid and the hollow shaft, we have in each case let D represent the external diameter of the shaft. To avoid confusion it will, in the following comparison, be convenient to let D_S represent the diameter of the solid shaft, D and d representing as before the external and the internal diameter respectively of the hollow shaft.

Let T_S = the torsional resistance of the solid shaft.
,, T_H = ,, ,, ,, hollow ,,
,, f_s = the shear stress in the material at the outer circumference.

Now,

$$T_S = \frac{\pi}{16} D_S^3 f_s,$$

and

$$T_H = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right) f_s.$$

$$\therefore \frac{T_H}{T_S} = \frac{\frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right) f_s}{\frac{\pi}{16} D_S^3 f_s} = \frac{D^4 - d^4}{D D_S^3}.$$

The weights of the shafts are proportional to their sectional areas, assuming their lengths to be the same, so that

$$\frac{\text{Weight of hollow shaft}}{\text{Weight of solid shaft}} = \frac{\frac{\pi}{4}(D^2 - d^2)}{\frac{\pi}{4} D_S^2} = \frac{D^2 - d^2}{D_S^2}.$$

As we are supposing the shafts to be of equal weight,

$$D_S^2 = D^2 - d^2.$$

Now

$$\frac{T_H}{T_S} = \frac{D^4 - d^4}{D D_S^3} = \frac{(D^2 - d^2)(D^2 + d^2)}{D D_S^3}.$$

Substituting for $(D^2 - d^2)$, the value D_S^2,

$$\frac{T_H}{T_S} = \frac{D_S^2 (D^2 + d^2)}{D D_S^3} = \frac{D^2 + d^2}{D D_S}.$$

From the relation, $D_S^2 = D^2 - d^2$

$$D_S = \sqrt{D^2 - d^2}.$$

Therefore, $$\frac{T_H}{T_S} = \frac{D^2 + d^2}{D\sqrt{D^2 - d^2}}.$$

Dividing numerator and denominator by D^2,

$$\frac{T_H}{T_S} = \frac{1 + \frac{d^2}{D^2}}{\frac{D}{D^2}\sqrt{D^2 - d^2}} = \frac{1 + \left(\frac{d}{D}\right)^2}{\frac{1}{D}\sqrt{D^2 - d^2}} = \frac{1 + \left(\frac{d}{D}\right)^2}{\sqrt{\frac{D^2 - d^2}{D^2}}}.$$

$$\therefore \frac{T_H}{T_S} = \frac{1 + \left(\frac{d}{D}\right)^2}{\sqrt{1 - \left(\frac{d}{D}\right)^2}}.$$

Suppose now the internal diameter of the hollow shaft to be one-half the external diameter; i.e. $\frac{d}{D} = \frac{1}{2}$.

Then, $$\frac{T_H}{T_S} = \frac{1 + (\frac{1}{2})^2}{\sqrt{1 - (\frac{1}{2})^2}} = \frac{1 + \cdot 25}{\sqrt{1 - \cdot 25}} = \frac{1 \cdot 25}{\sqrt{\cdot 75}} = 1 \cdot 445.$$

Thus, a hollow shaft, the internal and external diameters of which are in the proportion of 1 to 2, is nearly one and a half times as strong as a solid shaft of the same length and weight.

121.—**Equivalent Solid Shaft.** It is sometimes convenient to determine the diameter of a solid shaft which would be of the same strength as a hollow shaft of the same material. Such a shaft is spoken of as an "equivalent solid shaft." The required diameter is easily determined as follows:—

Let D_S = diameter of equivalent solid shaft.
,, D = external diameter of hollow shaft.
,, d = internal ,, ,, ,,

As the shafts are to be of the same strength, their torsional resistances must be equal.

Torsional resistance of solid shaft $= \frac{\pi}{16} D_S^3 f_s$.

,, ,, hollow ,, $= \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right) f_s$.

Equating, $$\frac{\pi}{16} D_S^3 f_s = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right) f_s,$$

from which, $$D_S = \sqrt[3]{\frac{D^4 - d^4}{D}} = \sqrt[3]{D^3 - \frac{d^4}{D}} = \sqrt[3]{D^3\left(1 - \frac{d^4}{D^4}\right)}.$$

$$\therefore D_S = D\sqrt[3]{1 - \left(\frac{d}{D}\right)^4}.$$

122.—**Variable Twisting Moment.** In many examples which occur in actual practice, the twisting moment exerted on a shaft is not uniform, but varies throughout each revolution. It is quite obvious that the shaft must be sufficiently strong to resist the greatest twisting moment to which it is subjected, and in calculating its strength, therefore, the torsional resistance must be equated to the maximum twisting moment.

An actual example of a shaft subjected to a variable twisting moment is the crank shaft of a steam (or internal combustion) engine. In such an engine, steam pressure acts on the piston, and the force resulting (or, more correctly, the greater portion of it) is transmitted through the piston rod and along the connecting rod to the crank pin, and, through the medium of the crank, rotates the crank shaft.

Now the turning moment in a case like this varies considerably throughout each revolution of the engine for the following reasons. In the first place, the effective pressure of the steam acting on the piston is not constant throughout the stroke, particularly after the point at which the supply of steam to the cylinder is cut off. In the second place, the effective leverage at which the force at the crank pin acts is also varying throughout each revolution. Again, the actual force due to the pressure is not all transmitted to the crank pin, a considerable portion of it being absorbed in accelerating the moving parts of the engine.

A correct determination of the real twisting moment necessitates a large amount of labour, such as estimating the weights of certain moving parts (so that the force required to accelerate these parts may be calculated) obtaining indicator diagrams from the engine and correcting these to allow for the accelerating forces, constructing twisting moment diagrams which show the actual twisting moment at every part of the revolution, and so on.

For all practical purposes, however, it is sufficient to calculate the twisting moment in the following manner :—

Let p=the maximum effective steam pressure on the piston, in lbs. per square inch.
,, d=the diameter of the piston, in inches.
,, l=the length of the crank ,,

Then,

$$\text{Total maximum effective load on piston} = p \times \frac{\pi}{4} \times d^2.$$

Effective leverage at which this force is supposed to be exerted=l.

$$\therefore \text{ Maximum twisting moment} = p \times \frac{\pi}{4} \times d^2 \times l.$$

This maximum twisting moment is equated to the torsional resistance of the shaft in order to get the diameter of shaft required.

123.—**Stiffness of Shafts.** The effect of a twisting moment applied to a shaft is illustrated by Fig. 60; thus, assuming the end, *A*, of the shaft to be fixed, a twisting moment applied to the other end, *B*, will deflect any line drawn parallel to the axis along the outer surface of the shaft, as indicated in the figure.

Now it is important in some cases that the amount of twist should not exceed a certain amount; in other words, the shaft should possess a certain amount of stiffness. In the case of light machinery, the required diameter of the shaft is often governed by its stiffness rather than by its strength, and we shall now deduce an expression for determining the diameter of a shaft when the stiffness is the governing factor.

Assuming the shaft of Fig. 60 to be made up of a large number of thin discs, then each disc, except the one at the extreme left, will be rotated slightly, the amount of rotation of each being slightly in excess of the disc on its immediate left. Consider now any two consecutive discs which we may suppose to be an indefinitely small distance apart. Prior to the application of the twisting moment, any straight line drawn through the centre of one of the discs would coincide with the corresponding straight line drawn through the centre of the other disc, but after the twisting moment had been applied, the two lines would make a small angle with each other.

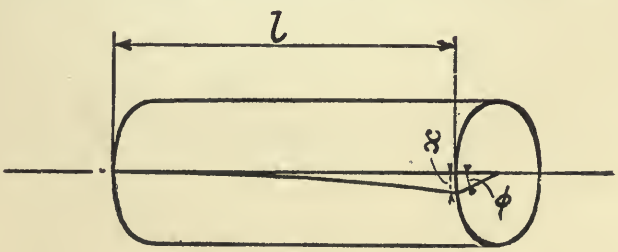

Fig. 63

Let D = the diameter of the shaft, in inches.
,, R = the radius ,, ,, ,,
,, l = the length ,, ,, ,,
,, f_s = the shear stress at the outer circumference of the shaft, in lbs. per square inch.
,, G = the modulus of rigidity of the shaft material, in lbs. per square inch.
,, T.M. = twisting moment, in pound-inches.

Let the very small distance between the two consecutive discs be represented by dl, and the angle between the two straight lines drawn through the centres of the discs by $d\phi$, dl being in inch units and $d\phi$ in circular measure, i.e. radians.

Now we know from previous work that

$$\text{Modulus of rigidity} = \frac{\text{Shear stress}}{\text{Shear strain}}.$$

The shear stress is f_s, and the shear strain is equal to surface twist divided by length. Thus, referring to Fig. 63,

$$\text{Shear strain} = \frac{x}{l} = \frac{R\phi}{l} = \frac{D\phi}{2l}.$$

Considering then a very short length of shaft, dl,

$$\text{Shear strain} = \frac{D}{2} \cdot \frac{d\phi}{dl}.$$

Hence,
$$G = \frac{f_s}{\frac{D}{2} \cdot \frac{d\phi}{dl}},$$

from which,
$$d\phi = \frac{f_s 2}{GD} \cdot dl.$$

Integrating,
$$\phi = \frac{f_s 2}{GD} \int dl.$$

The length of the shaft is l, so we integrate between the limits 0 and l. Thus,

$$\phi = \frac{f_s 2}{GD} \int_0^l dl = \frac{f_s 2}{GD} \cdot l = \frac{2lf_s}{GD}.$$

Now,
$$\text{T.M.} = \frac{\pi}{16} D^3 f_s,$$

from which,
$$f_s = \frac{\text{T.M}}{\frac{\pi}{16} D^3}.$$

Substitute this value of f_s in the relation,

$$\phi = \frac{2lf_s}{GD};$$

then
$$\phi = \frac{2l\text{T.M.}}{GD\frac{\pi}{16}D^3} = \frac{32}{\pi} \frac{l\text{T.M.}}{GD^4}.$$

This expression gives the value of ϕ in radians, and the value in degrees is obtained by multiplying by 57·3. Thus,

$$\phi = \frac{32}{\pi} \frac{l\text{T.M.}}{GD^4} \times 57 \cdot 3 = \frac{584 l\text{T.M.}}{GD^4}.$$

$$\therefore\ \phi = \frac{584 l\text{T.M.}}{GD^4}, \text{ and T.M.} = \frac{\phi GD^4}{584 l}.$$

We see from this that the stiffness of a solid round shaft varies as the fourth power of its diameter, whereas the strength varies as the cube of the diameter.

For a hollow shaft of external diameter D and internal diameter d,

$$\phi=\frac{584l\text{T.M.}}{\text{G}(\text{D}^4-d^4)}, \text{ and T.M.}=\frac{\phi\text{G}(\text{D}^4-d^4)}{584l}.$$

Summing up, the strength of round shafts is obtained from the relations,

$$\text{T.M.}=\frac{\pi}{16}\text{D}^3f_s \text{ (for solid shafts);}$$

$$\text{T.M.}=\frac{\pi}{16}\left(\frac{\text{D}^4-d^4}{\text{D}}\right)f_s \text{ (for hollow shafts);}$$

whilst the stiffness is obtained from the relations,

$$\text{T.M.}=\frac{\phi\text{GD}^4}{584l} \text{ (for solid shafts);}$$

$$\text{T.M.}=\frac{\phi\text{G}(\text{D}^4-d^4)}{584l} \text{ (for hollow shafts).}$$

124.—The formula for the stiffness of a solid shaft, viz.

$$\text{T.M.}=\frac{\phi\text{GD}^4}{584l}$$

is the one generally used by land engineers. In marine work, however, the **Board of Trade use the formula,**

$$\text{T}=\frac{140d^4a}{\text{L}},$$

where T = twisting moment, in pound-feet;
d = diameter of shaft, in inches;
a = angle of twist, in degrees;
L = length of shaft, in feet.

In the formula deduced above, all the dimensions are in inch units, the load or force units being in pounds; hence the twisting moment is expressed in pound-inches. Also, the angle of twist, which we call ϕ, is measured in radians, whereas the Board of Trade measure the angle in degrees and call it a.

To make the two expressions for the twisting moment correspond, substitute a for ϕ. Then,

$$\text{Twisting moment, in pound-inches}=\frac{a\text{GD}^4}{584l},$$

$$\text{Twisting moment, in pound-feet}=\frac{a\text{GD}^4}{584l\times 12},$$

where l is the length in inches.

In the Board of Trade formula, the length is taken in feet, whilst we have taken it in inches.

The expression for the twisting moment in pound-feet, if the length be given in feet, will then be,

$$\text{T.M.}=\frac{aGD^4}{584\times 12\times 12\times L}.$$

The value of G is about 12,000,000 lbs. per square inch.

Substituting this value,

$$\text{T.M.}=\frac{a\times 12{,}000{,}000D^4}{584\times 12\times 12\times L}=\frac{142{\cdot}6D^4a}{L}.$$

As an approximation, the Board of Trade take the final expression as

$$\text{T.M.}=\frac{140d^4a}{L}\text{ for solid shaft,}$$

and $$\text{T.M.}=\frac{140a(D^4-d^4)}{L}\text{ for hollow shaft.}$$

An alternative formula connecting the twisting moment, the moment of inertia, the stress in the material, and the angle of twist of a shaft is the following:—

$$\frac{TM}{J}=\frac{\phi G}{l}=\frac{f_s}{R},$$

where T.M.=twisting moment, in pound-inches;
ϕ=angle of twist;
G=modulus of rigidity of the shaft material;
J=moment of inertia of shaft;
l=length of shaft;
R=radius of shaft.

To deduce this equation, proceed exactly on the same lines as before up to the point where we saw that,

$$\text{Shear strain}=\frac{R\phi}{l}.$$

Now,

$$\frac{\text{Shear stress}}{\text{Shear strain}}=\text{modulus of rigidity, G,}$$

and, $$\text{Shear strain}=\frac{R\phi}{l}.$$

Substituting, $$\frac{f_s}{\frac{R\phi}{l}}=G.$$

Hence, $$f_s=\frac{R\phi}{l}\times G.$$

Dividing throughout by R,

$$\frac{f_s}{R}=\frac{\phi}{l}\times G \quad . \quad . \quad . \quad (1)$$

Again, $\quad$ Force = stress × area.

Moment = force × leverage at which force acts.

$\therefore$ Moment = stress × area × R.

But $\quad$ Stress = strain × G,

and $\quad$ $\text{Strain}=\frac{R\phi}{l}.$

$$\therefore \text{Stress}=\frac{R\phi}{l}\times G.$$

$$\text{Moment}=\frac{R\times\phi}{l}\times G\times \text{area}\times R,$$

and the total $\quad$ $\text{T.M.}=\frac{\phi}{l}\times G\times\Sigma(\text{area}\times R^2).$

Now the sum of all such quantities as (area × R^2) represents the polar moment of inertia of the shaft.

Hence, $\text{T.M.}=\frac{\phi}{l}\times G\times J$, where J represents the polar moment of inertia.

$$\therefore \text{T.M.}=\frac{\phi}{l}\times G\times J,$$

and $\quad$ $\frac{TM}{J}=\frac{\phi}{l}\times G \quad . \quad . \quad . \quad (2)$

From (1), $\frac{\phi}{l}\times G$ also equals $\frac{f_s}{R}$.

$$\therefore \frac{\text{T.M.}}{J}=\frac{\phi G}{l}=\frac{f_s}{R},$$

a formula connecting twisting moment, moment of inertia, stress in the material, and angle of twist.

125.—**Shafts subjected to both Bending and Twisting.** It has been assumed in the foregoing considerations of the strength of shafts that the shafts have been subjected to torsion only. In actual practice, however, it frequently happens that a shaft is exposed also to a bending action due to the weight of pulleys, couplings, etc. In many instances the bending is of little importance, and need not be taken into account in calculating the strength of the shaft, but in others it cannot be neglected. Thus, an engine crank shaft, in addition to being exposed to a twisting moment, has usually to support the weight of a heavy flywheel, which gives rise to a more or less severe bending action. The shaft is in consequence subjected to stresses due both to twisting and bending, and we must next consider how the strength is calculated under such conditions.

In Chapter IV. we considered the action of a shear stress in combination with a tensile or a compressive stress, and showed how to find an equivalent stress which would balance the combined stresses.

Letting f_e represent the equivalent tensile stress, f_s the shear stress, and f_t the tensile stress, we deduced the following relation,

$$f_e=\frac{f_t}{2}+\sqrt{f_s^2+\frac{f_t^2}{4}}.$$

From the relation which gives us the strength of a shaft to resist torsion, the value of f_s in terms of the twisting moment, T.M., can be found, and from another expression which gives us the strength of a beam (or shaft) of circular section to resist bending, we can find the value of f_t in terms of the bending moment, which may be denoted by B.M. By substituting these values of f_s and f_t in the above expression for f_e, and rearranging, we can obtain an expression which gives us an equivalent twisting moment in terms of the twisting moment and the bending moment. By equating this equivalent twisting moment to the torsional resistance of the shaft, the diameter of shaft required to resist a combined twisting and bending moment may then be calculated.

Now for a solid shaft,

$$\text{T.M.}=\frac{\pi}{16}D^3 f_s,$$

where D is the diameter of the shaft.

As yet, the student is not supposed to have any knowledge of bending moments and the strength of beams, so he will, for the present, have to accept the statement that the strength of a beam of circular section to resist bending is given by the relation,

$$\text{B.M.}=\frac{\pi}{32}D^3 f_t,$$

where B.M. represents the bending moment acting on the beam, and f_t the maximum tensile stress due to the bending. Now from the equations for T.M. and B.M.,

$$f_s=\frac{\text{T.M.}}{\frac{\pi}{16}D^3}$$

and

$$f_t=\frac{\text{B.M.}}{\frac{\pi}{32}D^3}.$$

It will be convenient to write Z for $\frac{\pi}{32}D^3$. Then.

$$f_s = \frac{T.M.}{2Z},$$

and $$f_t = \frac{B.M.}{Z}.$$

Now substitute these values of f_s and f_t in the expression for f_e;

$$f_e = \frac{f_t}{2} + \sqrt{f_s^2 + \frac{f_t^2}{4}}$$

$$= \frac{B.M.}{2Z} + \sqrt{\left(\frac{T.M.}{2Z}\right)^2 + \frac{\left(\frac{B.M.}{Z}\right)^2}{4}}$$

$$= \frac{B.M.}{2Z} + \sqrt{\frac{T.M.^2}{4Z^2} + \frac{B.M.^2}{4Z^2}}$$

$$= \frac{B.M.}{2Z} + \frac{1}{2Z}\sqrt{T.M.^2 + B.M.^2}$$

$$= \frac{1}{2Z}(B.M. + \sqrt{T.M.^2 + B.M.^2}).$$

Hence, $$f_e Z = \frac{B.M. + \sqrt{T.M.^2 + B.M.^2}}{2}.$$

Now $f_e Z$ represents the equivalent bending moment, which may be denoted by B.M.$_e$, so

$$B.M._e = \frac{B.M. + \sqrt{T.M.^2 + B.M.^2}}{2}.$$

Again, from the above,

$$f_e 2Z = B.M. + \sqrt{T.M.^2 + B.M.^2}.$$

But $f_e 2Z$ represents the equivalent twisting moment, which may be denoted by T.M.$_e$, so we have, finally,

$$\underline{T.M._e = B.M. + \sqrt{T.M.^2 + B.M.^2}}.$$

This relation is very useful in cases where a shaft is subject both to twisting and bending. The twisting and the bending moments are each calculated separately, the equivalent twisting moment being next calculated from the relation referred to. The latter is then equated to the torsional resistance of the shaft. It is to be noted that the stress due to the equivalent twisting moment is a tensile stress, and not a shear stress, as in the case where the shaft is subjected to a twisting moment only.

In many examples to which this relation applies, it will be found that the values of T.M. and B.M. are large, often millions of pound-inches, and the squaring of such large numbers is cumbersome. To avoid this, the following simple construction may be made use of.

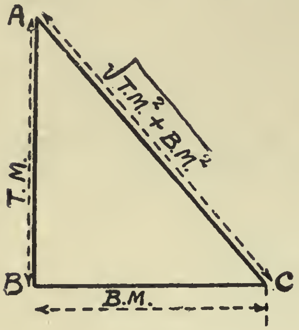

Fig. 64

Draw a triangle, ABC (Fig. 64), making one side, AB, say, equal in length to the T.M. to any convenient scale, and the other side, BC, equal to the B.M. to the same scale. Then the length of the hypotenuse, to the same scale, gives the value of $\sqrt{T.M.^2+B.M.^2}$.

This, of course, follows from the forty-seventh Proposition of Euclid, Book I., which states that "in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides." Thus, in Fig. 64,

$$AB^2+BC^2=AC^2,$$

from which,
$$AC=\sqrt{AB^2+BC^2}.$$

126.—**Strength of Shaft Coupling Bolts.** It is frequently necessary to couple together two lengths of shafting, for which purpose shaft couplings are employed. For small shafts, the couplings are usually distinct pieces keyed to the ends of the shafts.

In the case of large shafts, however, such as propeller shafts, engine crank shafts, etc., solid couplings are generally employed. Thus, the ends of the shafts are each provided with a solid flange which forms part of the shaft, and the two flanges are then fastened together by means of a number of bolts.

The bolts of a solid flange coupling have to transmit the whole of the power passing through the shafts, and it is important to consider how the required diameter of the bolts may be determined in any particular case.

Let $T.R._S$=torsional resistance of shaft.
,, $T.R._B$= ,, ,, bolts.
,, D_S=diameter of solid shaft.
,, d_b=diameter of bolts in coupling.
,, n=number ,, ,,
,, r_c=radius of bolt centre circle.
,, f_s=shear stress in shaft and bolt material.

The bolts are required to resist the same twisting moment as the shaft. Therefore,

$$T.R._B=T.R._S.$$

The torsional resistance of the bolts is clearly equal to

$$n\times\frac{\pi}{4}d_b^2\times f_s\times r_c,$$

assuming them to be a perfect fit. If, however, there be any clearance, the bolts will not be subject to pure shear, and the torsional resistance must be taken as only three-quarters of the above, for the reasons explained in connection with suspension links in the previous chapter.

(We may assume the bolts and the shafts both to be made of the same material.)

Hence, $$T.R._B = n\frac{\pi}{4}d_b^2 f_s r_c \times \frac{3}{4}.$$

Now for a solid shaft,

$$T.R._S = \frac{\pi}{16}D_S^3 f_s,$$

and $$T.R._B = T.R._S,$$

Then, $$n\frac{\pi}{4}d_b^2 f_s r_c \times \frac{3}{4} = \frac{\pi}{16}D_S^3 f_s,$$

from which, $$d_b^2 = \frac{D_S^3}{3nr_c}.$$

$$\therefore\ d_b = \sqrt[2]{\frac{D_S^3}{3nr_c}} \text{ (for solid shaft).}$$

If the shaft be hollow instead of solid, the external and internal diameters being D and d respectively, the torsional resistance of the shaft is

$$\frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)f_s,$$

and this is equated to the torsional resistance of the bolts, thus,

$$n\frac{\pi}{4}d_b^2 f_s r_c \times \frac{3}{4} = \frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)f_s.$$

Then, $$d_b^2 = \frac{D^4-d^4}{D} \div 3nr_c = \frac{D^4-d^4}{3nr_cD}.$$

$$\therefore\ d_b = \sqrt{\frac{D^4-d^4}{3nr_cD}} \text{ (for hollow shaft).}$$

BOARD OF TRADE FORMULÆ FOR MARINE SHAFTING

127.—**The Board of Trade formula for Coupling Bolts** in propeller shafting is :—

$$d_b = \sqrt{\frac{D_s^3}{4.n.l}},$$

where d_b=diameter of coupling bolts; n=number of bolts;
D_S= ,, ,, shaft; l=radius of bolt circle.

It differs from the above formula, owing to the fact that the coupling bolts are assumed to be a perfect fit, so that the bolts are subject to pure shear. Hence we neglect the $\frac{3}{4}$ introduced above. Equating the moment of resistance of the bolts to the moment of resistance of the shaft,

$$\frac{\pi}{16}D_s{}^3 f_s = \frac{\pi}{4} d_b{}^2 n l f_s,$$

$$d_b{}^2 = \frac{D_s{}^3}{4.n.l},$$

$$\therefore\ d_b = \sqrt{\frac{D_s{}^3}{4.n.l}}\ \text{(for solid shaft).}$$

For hollow shafts,

$$d_b = \sqrt{\frac{(D^4 - d^4)}{4.n.l.D}},$$

where D=outer diameter of shaft;
d=inner diameter.

128.—A number of general formulæ in connection with marine shafting are given by the Board of Trade to meet certain cases. Thus, for compound condensing engines with two or more cylinders, when the cranks are not overhung, the following formula is given :—

$$S = \sqrt[3]{\frac{C \times P \times D^2}{f\left(2 + \frac{D^2}{d^2}\right)}},$$

where S=diameter of shaft, in inches;
d^2=square of diameter of high pressure cylinder, in inches, or sum of squares of diameters when there are two or more high pressure cylinders;
D^2=square of diameter of low-pressure cylinder, in inches, or sum of squares of diameters when there are two or more low-pressure cylinders;
P=absolute pressure, in lbs. per square inch, that is, boiler pressure plus 15 lbs;
C=length of crank, in inches;
f=constant from the table which follows.

For ordinary condensing engines with one, two, or more cylinders, when the cranks are not overhung :—

$$S = \sqrt[3]{\frac{C \times P \times D^2}{3 \times f}},$$

where S=diameter of shaft, in inches;
D^2=square of diameter of cylinder, in inches, or sum of squares of diameters when there are two or more cylinders;
P=absolute pressure, in lbs. per square inch, that is, boiler pressure plus 15 lbs.;
C=length of crank, in inches;
f=constant from following table.

Angle between Cranks.	For Crank and Thrust Shafts.		For Tunnel Shafts.	For Propeller Shafts.
For two Cranks.				
90°	For paddle engines of the direct-acting type multiply constant in this column suitable for angles of cranks by 1·4.	1047	1221	890
100°		966	1128	821
110°		904	1055	768
120°		855	997	727
130°		817	953	694
140°		788	919	670
150°		766	894	651
160°		751	877	638
170°		743	867	631
180°		740	864	629
For three Cranks.				
120°		1110	1295	943

When there is only one crank, the constants applicable are those in the table opposite 180 degrees.

Note.—When the diameter of the crank shaft has been ascertained by calculation, the diameter of the propeller shaft may be found by multiplying the diameter of the crank shaft by 1·056, and that of tunnel shafts by multiplying by ·95.

For turbine engines :—

$$S=\sqrt{\frac{I.H.P.\times f}{R}},$$

where S=diameter of shaft, in inches ;

I.H.P.=estimated maximum indicated horse power transmitted through shaft ;

R=number of revolutions per minute ;

f=60·3 for tunnel shafts ;

f=82·8 for propeller shafts.

Note.—When the diameter of the tunnel shaft has been ascertained by calculation, the diameter of the propeller shaft may be found by multiplying the diameter of the tunnel shaft by 1·112.

These formulæ are based on Mr. M'Farlane Gray's approximation of mean referred pressure, which is :—

$$\text{Mean referred pressure}=\left(\frac{P}{1+\cdot 3R}\right),$$

where P=absolute pressure, in lbs. per square inch ;

R=ratio of expansion in engine.

Take, for example, a single engine. The Board of Trade formula in this case is,

$$S=\sqrt[3]{\frac{C\times P\times D^2}{3\times f}}.$$

Assuming the cut-off takes place at $\frac{6}{10}$ of the stroke, then $R=\frac{10}{6}$, and

$$\frac{P}{(1+{\cdot}3R)}=\frac{P}{(1+\frac{3}{10}\times\frac{10}{6})}=\frac{P}{1+\frac{1}{2}}=\frac{2P}{3} \quad . \quad . \quad . \quad (1)$$

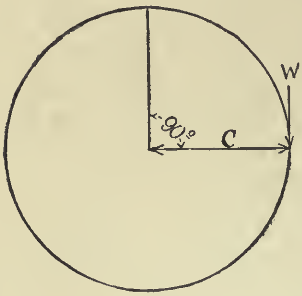

Fig. 65

In the case of a single crank, the position of maximum twisting moment may be assumed to occur when the crank is at an angle of 90 degrees to the axis of the cylinder.

The twisting moment will be the product of *W*, the weight on the piston, and *C*, the length of the crank.

Thus, T.M.$=$W$\times$C. (See Fig. 65.)

W, the weight on the piston$=$area of piston$\times$mean pressure.

$$\therefore \text{ T.M.}=\frac{\pi D^2}{4}\times P\times C.$$

But from (1), the mean pressure is$=\frac{2P}{3}$.

$$\therefore \text{ T.M.}=\frac{C\times P\times \pi D^2\times 2}{4\times 3}.$$

Equating this to the moment of resistance of the shaft,

$$\frac{\pi}{16}S^3 f_s=\frac{C\times P\times D^2\times \pi\times 2}{4\times 3},$$

and $$S^3=\frac{C\times P\times D^2\times \cancel{\pi}\times 2\times 16}{4\times 3\times \cancel{\pi}\times f_s},$$

from which, $$S=\sqrt[3]{\frac{C\times P\times D^2\times 8}{3\times f_s}}.$$

Comparing this with the Board of Trade formula,

$$S=\sqrt[3]{\frac{C\times P\times D^2}{3\times f}},$$

we have $\frac{8}{f_s}$ in place of $\frac{1}{f}$, and from the table, the constant f in this case is 740.

Therefore, $$\frac{8}{f_s}=\frac{1}{740},$$

and $$f_s=740\times 8=\underline{5920}.$$

It will be seen, therefore, that the Board of Trade allow a stress of 5920 lbs. per square inch on the shaft.

129.—Next take the case of a compound engine, say with two cranks at 90 degrees.

In this case, the weight W is split up between the two pistons; in other words, there is a load of $\frac{W}{2}$ on each crank, and the position of maximum twisting moment will be as shown in Fig. 66. Hence, the twisting moment will be,

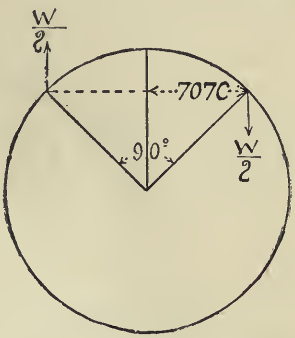

Fig. 66

$$\text{T.M.}=\frac{W}{2}\times C\sin 45^\circ+\frac{W}{2}\times C\sin 45^\circ$$
$$=2\left(\frac{W}{2}\times C\sin 45^\circ\right)$$
$$=W\times C\times\cdot 7071.$$

Again, cutting off at $\frac{6}{10}$ of the stroke in the H.P. cylinder, and the ratio between the L.P. cylinder and the H.P. cylinder being $\frac{D^2}{d^2}$, the ratio of expansion,

$$R=\frac{10}{6}\times\frac{D^2}{d^2}.$$

Using M'Farlane Gray's approximation for mean referred pressure, then :—

$$\frac{P}{1+\cdot 3R}=\frac{P}{1+\left(\frac{3}{10}\times\frac{10}{6}\times\frac{D^2}{d^2}\right)}=\frac{P}{1+\frac{1}{2}\frac{D^2}{d^2}}=\frac{2P}{2+\frac{D^2}{d^2}},$$

and twisting moment, T.M.$=\cdot 7071$WC.

Also,
$$W=\frac{\pi d^2}{4}\times\frac{2P}{2+\frac{D^2}{d^2}}.$$

$$\therefore\quad \text{T.M.}=\frac{\cdot 7071\times C\times\pi\times d^2\times 2P}{4\times\left(2+\frac{D^2}{d^2}\right)}$$
$$=\frac{CPd^2\pi\times\cdot 7071}{2\left(2+\frac{D^2}{d^2}\right)}.$$

Equating this to the moment of resistance of the shaft,

$$\frac{CPd^2\times\pi\times\cdot 7071}{2\times\left(2+\frac{D^2}{d^2}\right)}=\frac{\pi}{16}S^3 f_s,$$

$$\therefore\ S=\sqrt[3]{\frac{CPd^2\times{\cdot}7071\times16}{2\left(2+\frac{D^2}{d^2}\right)f_s}}$$

$$=\sqrt[3]{\frac{CPd^2\times{\cdot}7071\times8}{f_s\left(2+\frac{D^2}{d^2}\right)}}.$$

The Board of Trade formula is,

$$S=\sqrt[3]{\frac{CPd^2}{f\left(2+\frac{D^2}{d^2}\right)}},$$

and comparing this with our result, we have $\frac{{\cdot}7071\times8}{f_s}$ instead of $\frac{1}{f}$, and the constant in this case (see table) is 1047. Hence,

$$\frac{{\cdot}7071\times8}{f_s}=\frac{1}{1047},$$

and $\quad f_s=1047\times8\times{\cdot}7071=\underline{5920\text{ lbs. per square inch.}}$

It will be seen, therefore, that in this formula the different values for the factor f limit the stress on the shaft to 5920 lbs. per square inch.

According to the Board of Trade rules, the constant used for screw propeller shafts may be multiplied by 1·4 to apply to formulæ for paddle engines of the direct-acting type.

Consider the case of a compound engine with cranks at 90 degrees. On referring to Fig. 66, it will be seen that the maximum twisting moment occurs when each crank is 45 degrees off the dead centre. The twisting moment is then

$$\frac{W}{2}\times C\sin45^\circ+\frac{W}{2}\times C\sin45^\circ=W\times C\sin45^\circ=W\times C\times{\cdot}707.$$

Now consider the case of a compound engine with a paddle wheel. In this case, we have two shafts and two paddles, and the maximum twisting moment will occur when the cranks are at right angles to the axis of the cylinder. Thus,

$$\frac{W}{2}\times C+\frac{W}{2}\times C=W\times C.$$

It follows then that the ratio of the twisting moments for a screw shaft and a paddle shaft is as ·707 : 1. Therefore, the paddle shaft must be stronger than the screw shaft in the ratio of 1 : ·707, or as 1·4 : 1. For this reason, we multiply the constant for screw shafts by 1·4 when applying the formula to find the diameter of a paddle shaft.

Worked Examples

130.—(1) A winding drum is 18 feet diameter and raises a load of 3 tons. What is the twisting moment exerted on the shaft in ton-feet and in pound-inches when the load is stationary?

Twisting moment=force applied×leverage.

The force applied is clearly equal to the weight raised, neglecting the weight of the rope, etc. The leverage is equal to the radius of the drum. Hence,

$$\text{Twisting moment} = 3\ (\text{tons}) \times 9\ (\text{feet}) = \underline{27 \text{ ton-feet.}}$$

The twisting moment measured in pound-inches will be

$$3 \times 2240 \times 9 \times 12 = \underline{725{,}760.}$$

$$\text{Twisting moment} = \underline{27 \text{ ton-feet.}}$$

$$\text{,,} \qquad \text{,,} \qquad = \underline{725{,}760 \text{ pound-inches.}}$$

(2) A rope driving pulley is 5 feet diameter and transmits 30 horse power at a speed of 120 revolutions per minute. What is the effective force exerted on the pulley rim? What is the twisting moment exerted on the shaft to which the pulley is keyed?

Let F=the effective force in lbs. acting at the pulley rim.

In one revolution of the pulley, the force may be supposed to pass through a distance equal to the circumference of the pulley, i.e. $\pi \times 5$ feet. Hence,

$$\text{Work done per revolution, in foot-pounds} = F \times \pi \times 5.$$

The pulley revolves 120 times per minute. Therefore,

$$\text{Work done per minute} = F \times \pi \times 5 \times 120.$$

The horse power transmitted is 30, which means that the number of foot-pounds of work transmitted by the pulley in one minute is $30 \times 33{,}000$. This work must be equal to the work done per minute by the force F, so that we have the equation,

$$F \times \pi \times 5 \times 120 = 30 \times 33{,}000.$$

$$\therefore\ F = \frac{30 \times 33{,}000}{\pi \times 5 \times 120} = \underline{525 \text{ lbs.}}$$

Effective force at pulley rim=$\underline{525 \text{ lbs.}}$

The twisting moment exerted on the shaft is equal to this force multiplied by its leverage, which is equal to the radius of the pulley. Therefore,

$$\text{Twisting moment} = 525 \times 2{\cdot}5 \times 12 = \underline{15{,}750 \text{ pound-inches.}}$$

We may, of course, obtain the twisting moment in pound-inches directly by using the relation,

$$\text{T.M.}=\frac{63{,}000 \text{ H.P.}}{\text{N}},$$

where H.P.=horse power transmitted;
N=speed of pulley or shaft, in revolutions per minute.

Thus, $$\text{T.M.}=\frac{63{,}000\times 30}{120}=\underline{15{,}750 \text{ pound-inches.}}$$

131.—(3) Determine the diameter of a mild steel shaft which is to transmit 80 horse power at a speed of 100 revolutions per minute. Allowable shear stress, 7000 lbs. per square inch.

The diameter is obtained from the relation,

$$\text{T.M.}=\frac{\pi}{16}\text{D}^3 f_s,$$

where T.M.=twisting moment, in pound-inches;
D=diameter of shaft, in inches;
f_s=allowable shearing stress, in lbs. per square inch.

It is first necessary to find the twisting moment, which is readily obtained from the relation,

$$\text{T.M.}=\frac{63{,}000 \text{ H.P.}}{\text{N}},$$

where H.P.=horse power to be transmitted;
N=speed of shaft, in revolutions per minute.

Substituting the known values,

$$\text{T.M.}=\frac{63{,}000\times 80}{100}=50{,}400 \text{ pound-inches.}$$

Equating this to the torsional resistance of the shaft, and substituting 7000 for f_s,

$$50{,}400=\frac{\pi}{16}\text{D}^3\times 7000.$$

$$\text{D}^3=\frac{50{,}400\times 16}{\pi\times 7000}=36{\cdot}7.$$

$$\text{D}=\sqrt[3]{36{\cdot}7}=3{\cdot}32 \text{ inches, or, say, } 3\tfrac{3}{8} \text{ inches}$$

Required diameter of shaft=$\underline{3\tfrac{3}{8}\text{ inches.}}$

(4) A shaft $3\frac{1}{2}$ inches diameter is subjected to a twisting moment of 3000 pound-feet. What is the maximum shear stress in the shaft?

The stress is obtained from the general relation,

$$\text{T.M.}=\frac{\pi}{16}\text{D}^3 f_s,$$

where T.M. = twisting moment, in pound-inches;
D = diameter of shaft, in inches;
f_s = shear stress in shaft, in lbs. per square inch.

Rearranging, $$f_s=\frac{\text{T.M.}\times 16}{D^3\times\pi}.$$

Note that the twisting moment must be reduced from pound-feet to pound-inches to suit the formula. Substituting the known values,

$$f_s=\frac{3000\times 12\times 16}{3\cdot 5\times 3\cdot 5\times 3\cdot 5\times\pi}=\underline{4290 \text{ lbs. per square inch.}}$$

Maximum shear stress = $\underline{\text{4290 lbs. per square inch.}}$

(5) A wrought-iron shaft 1 inch diameter is broken by a load of 800 lbs. applied to the end of a 12-inch lever keyed to the shaft. Find what force must be applied to the end of a lever 15 inches long to break a 3-inch shaft of the same material.

The strength of a solid round shaft varies as the cube of its diameter. The relative strengths of a 1-inch and a 3-inch shaft are therefore as $1^3 : 3^3$, or as $1 : 27$.

The 3-inch shaft is consequently able to resist a twisting moment twenty-seven times as great as that which a 1-inch shaft could resist.

Let T.M._1 be the twisting moment which breaks the 1-inch shaft.
,, T.M._3 ,, ,, ,, ,, ,, 3-inch ,,

Then, $$\text{T.M.}_1 : \text{T.M.}_3 :: 1 : 27.$$

Now, $$\text{T.M.}_1 = 800\times 12 \text{ pound-inches.}$$
$$\text{T.M.}_3 = F\times 15 \quad ,, \quad ,,$$

where F = the force applied to the lever on the 3-inch shaft.

$$\therefore\ 800\times 12 : F\times 15 :: 1 : 27.$$

$$F\times 15=\frac{800\times 12\times 27}{1}$$

$$F=\frac{800\times 12\times 27}{15}=\underline{17{,}280 \text{ lbs.}}$$

Force required = $\underline{\text{17,280 lbs.}}$

132.—(6) A shaft transmits 800 horse power at a speed of 60 revolutions per minute to drive a number of machines. Supposing several additional machines are to be put down, requiring another 200 horse power to drive them, what would you do to obtain the increased power?

The increased power may be obtained simply by increasing the speed of the shaft proportionally, as this does not affect in any way the strength of the shaft, the twisting moment, which really governs the diameter of shaft required for the work, remaining unaltered, however much the speed be altered. We have then a simple proportion sum to solve, thus:—If a shaft transmits 800 horse power at a speed of 60 revolutions per minute, at what speed must it be run to transmit 800+200=1000 horse power? Stating the question as a proportion,

$$800 : 60 :: 1000 : x.$$

$$x = \frac{60 \times 1000}{800} = \underline{75}.$$

∴ Speed of shaft must be increased to 75 revolutions per minute.

(7) Assuming a 3-inch shaft is capable of transmitting safely 50 horse power when running at a speed of 100 revolutions per minute, what power could be safely transmitted by a 5-inch shaft of the same material running at a speed of 75 revolutions per minute?

We shall first find what horse power a 3-inch shaft would transmit when running at the speed of the second shaft, viz. 75 revolutions per minute. Thus, if 50 horse power is transmitted at a speed of 100 revolutions per minute, what will be the power transmitted if the shaft only run at 75 revolutions? Stating as a simple proportion, we have,

$$50 : 100 :: x : 75.$$

$$x = \frac{50 \times 75}{100} = 37{\cdot}5.$$

A 3-inch shaft will consequently transmit 37·5 horse power at a speed of 75 revolutions per minute, and we have now to find what power could be transmitted by a 5-inch shaft running at the same speed.

Let $T.M._3$ = the twisting moment exerted on the 3-inch shaft.
,, $T.M._5$ = ,, ,, ,, ,, 5-inch ,,
,, $H.P._3$ = power transmitted by 3-inch shaft.
,, $H.P._5$ = ,, ,, 5-inch ,,
,, N_3 = speed in revolutions per minute of 3-inch shaft.
,, N_5 = ,, ,, ,, 5-inch ,,

The relative strengths of a 3-inch and a 5-inch shaft of the same material are as $3^3 : 5^3$, or as $27 : 125$, i.e. $1 : 4{\cdot}63$. A 5-inch shaft is, therefore, 4·63 times as strong as a 3-inch shaft, or will withstand a twisting moment 4·63 times as great as will a 3-inch shaft. Hence,

$$T.M._3 = \frac{T.M._5}{4{\cdot}63}.$$

Now, $$\text{T.M.}_3 = \frac{63{,}000\ \text{H.P.}_3}{N_3},$$

and $$\text{T.M.}_5 = \frac{63{,}000\ \text{H.P.}_5}{N_5}.$$

$$\therefore\ \frac{63{,}000\ \text{H.P.}_3}{N_3} = \frac{63{,}000\ \text{H.P.}_5}{4{\cdot}63\ N_5}.$$

We are now supposing both the shafts to be running at 75 revolutions per minute, so we can cancel out N_3 and N_5. The 63,000 may also be cancelled out, and we are left with

$$\text{H.P.}_3 = \frac{\text{H.P.}_5}{4{\cdot}63}.$$

Substituting 37·5 for H.P.$_3$, we have,

$$37{\cdot}5 = \frac{\text{H.P.}_5}{4{\cdot}63},$$

and $$\text{H.P.}_5 = 37{\cdot}5 \times 4{\cdot}63 = 173{\cdot}7.$$

The 5-inch shaft running at 75 revolutions per minute is thus capable of transmitting safely 173·7 horse power.

The foregoing question might be worked out very readily by remembering that *the power a shaft is capable of transmitting, at any particular speed, as well as the strength of the shaft, varies as the cube of its diameter,* but the detailed working has been given with the object of showing exactly why this is so.

133.—(8) A hollow shaft is 13 inches diameter externally and 7 inches internally. It transmits 3000 horse power at a speed of 70 revolutions per minute. What is the maximum stress in the shaft?

The general relation for the strength of a hollow shaft is,

$$\text{T.M.} = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right) f_s,$$

where T.M. = twisting moment, in pound-inches;
D = external diameter of shaft, in inches;
d = internal ,, ,, ,,
f_s = maximum shear stress, in lbs. per square inch.

First find the twisting moment on the shaft from the relation

$$\text{T.M.} = \frac{63{,}000\ \text{H.P.}}{N},$$

where H.P. = horse power transmitted;
N = speed of shaft, in revolutions per minute.

Substituting the given data,

$$\text{T.M.}=\frac{63{,}000\times3000}{70}.$$

Now equate this to the torsional resistance of the shaft, substituting for D and d their respective values.

$$\frac{63{,}000\times3000}{70}=\frac{\pi}{16}\left(\frac{13^4-7^4}{13}\right)f_s.$$

Then,
$$f_s=\frac{63{,}000\times3000\times16\times13}{70\times\pi(13^4-7^4)}$$
$$=\frac{900\times3000\times16\times13}{\pi\times26{,}160}$$
$$=6830 \text{ lbs. per square inch.}$$

∴ Maximum stress in shaft=6830 lbs. per square inch.

(9) A hollow shaft is required to transmit 6000 horse power at a speed of 90 revolutions per minute. The twisting moment is variable, the maximum being $1\frac{3}{4}$ times the mean. Determine the external and the internal diameters of the shaft, assuming the internal diameter is to be one-half the external. The shaft is of mild steel having an ultimate shear stress of 24 tons per square inch, and a factor of safety of eight is to be allowed.

To solve this problem, we again use the relation,

$$\text{T.M.}=\frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)f_s,$$

where T.M.=maximum twisting moment, in pound-inches;
D=external diameter of shaft, in inches;
d=internal " " "
f_s=safe shear stress, in lbs. per square inch.

It is first necessary to find the mean twisting moment exerted on the shaft, and afterwards multiply this by $1\frac{3}{4}$ in order to obtain the maximum twisting moment, as the shaft must be sufficiently strong to resist the latter.

The expression, $\frac{63{,}000 \text{ H.P.}}{N}$, where H.P.=horse power transmitted and N=speed of shaft in revolutions per minute, gives us the mean twisting moment. Substituting the values of H.P. and N,

$$\text{Mean T.M.}=\frac{63{,}000\times6000}{90}.$$

$$\text{Maximum T.M.}=\frac{63{,}000\times6000\times1{\cdot}75}{90}.$$

This maximum twisting moment must now be equated to the torsional resistance of the shaft. Thus,

$$\frac{63{,}000\times6000\times1{\cdot}75}{90}=\frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)f_s.$$

Both D and d are unknown, but as the question states that the internal diameter is to be one-half the external, we can substitute $\frac{D}{2}$ for d.

As for f_s, this is obtained by dividing the ultimate shear strength of the shaft material by the factor of safety. Thus,

$$\text{Safe stress}=\frac{\text{ultimate stress}}{\text{factor of safety}}=\frac{24}{8}=3 \text{ tons per square inch.}$$

Note that the twisting moment is measured in pound and inch units, and the stress must therefore be measured in pounds per square inch. Hence,

$$\frac{63{,}000\times6000\times1{\cdot}75}{90}=\frac{\pi}{16}\left\{\frac{D^4-\left(\frac{D}{2}\right)^4}{D}\right\}3\times2240.$$

$$\frac{D^4-\frac{D^4}{16}}{D}=\frac{63{,}000\times6000\times1{\cdot}75\times16}{90\times\pi\times3\times2240}.$$

$$\frac{15}{16}D^3=\frac{63{,}000\times6000\times1{\cdot}75\times16}{90\times\pi\times6720}.$$

$$D^3=\frac{63{,}000\times6000\times1{\cdot}75\times16\times16}{90\times\pi\times6720\times15}$$

$$=5940.$$

$$\therefore\ D=\sqrt[3]{5940}=\underline{18{\cdot}2 \text{ inches.}}$$

External diameter of shaft $=\underline{18{\cdot}2 \text{ inches.}}$

Internal ,, ,, $=\underline{9{\cdot}1 \text{ inches.}}$

134.—(10) What percentage reduction of strength results from boring a 5-inch hole through a 12-inch shaft? What is the reduction in weight?

The strength of a shaft is measured by its torsional resistance, which, for a solid shaft, is $\frac{\pi}{16}D_S{}^3f_s$ and for a hollow shaft is $\frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)f_s$, where D_S is the diameter of the solid shaft, D and d the external and internal diameters respectively of the hollow shaft, and f_s the shearing stress in the material. The relative strengths are, therefore, as

$$\frac{\pi}{16}D_S{}^3 f_s : \frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)f_s, \text{ or as } D_S{}^3 : \frac{D^4-d^4}{D},$$

since $\frac{\pi}{16}$ and f_s are common to both expressions.

For the case in point, D_S is the same as D, and the relative strengths of the shaft as originally made, i.e. solid, and after boring the hole, will thus be as $12^3 : \frac{12^4-5^4}{12}$, or as $1728 : 1676$, i.e. $1 : {\cdot}97$. Thus, after boring a 5-inch hole through the shaft, the strength is still ·97 of what it was originally.

Percentage reduction of strength $=\frac{1-{\cdot}97}{1}\times 100=\frac{{\cdot}03}{1}\times 100=\underline{3{\cdot}0}.$

The weight of the shaft varies as its sectional area, the length remaining the same.

$$\text{Sectional area originally} \quad =\frac{\pi}{4}D_S{}^2.$$

$$\text{,,} \qquad \text{,,} \qquad \text{after boring} =\frac{\pi}{4}(D^2-d^2).$$

The areas or the weights are therefore as $\frac{\pi}{4}D_S{}^2 : \frac{\pi}{4}(D^2-d^2)$ or as $\frac{\pi}{4}\times 12^2 : \frac{\pi}{4}(12^2-5^2)$, i.e. as $144 : 119$, or $1 : {\cdot}827$. Thus, after boring, the weight of the shaft is only ·827 of what it was originally.

Percentage reduction of weight $=\frac{1-{\cdot}827}{1}\times 100=\frac{{\cdot}173}{1}\times 100=17{\cdot}3.$

This example serves to show that whilst the weight of the shaft is considerably reduced by boring a hole through it, the strength is reduced only very slightly, and thus great saving of metal is effected, with no material loss of strength.

(11) What are the relative strengths of a hollow and a solid shaft of the same weight, the internal diameter of the hollow shaft being ·375 of the external diameter?

We showed in the text that if T_H and T_S represent respectively the torsional resistances of a hollow and a solid shaft, d and D representing respectively the internal and external diameters of the hollow shaft, then,

$$\frac{T_H}{T_S}=\frac{1+\left(\frac{d}{D}\right)^2}{\sqrt{1-\left(\frac{d}{D}\right)^2}}.$$

In the present question, $d = \cdot 375D$. Hence,

$$\frac{T_H}{T_S} = \frac{1+(\cdot 375)^2}{\sqrt{1-(\cdot 375)^2}} = \frac{1+\cdot 141}{\sqrt{1-\cdot 141}} = \frac{1\cdot 141}{\cdot 925} = \underline{1\cdot 234.}$$

The torsional resistance of the hollow shaft is thus 1·234 times that of the solid shaft, so that the hollow shaft is 1·234 (or practically $1\frac{1}{4}$) times as strong as the solid shaft.

(12) A hollow shaft is 14 inches diameter externally and 8 inches internally. What should be the diameter of a solid shaft of the same material for equal strength?

We have here to find the equivalent solid diameter, which, as was shown in the text, is given by the relation,

$$D_S = D\sqrt[3]{1-\left(\frac{d}{D}\right)^4},$$

where D_S = diameter of solid shaft;
D and d = external and internal diameters respectively of the hollow shaft.

Substituting 14 and 8 respectively for D and d,

$$D_S = 14\sqrt[3]{1-\left(\frac{8}{14}\right)^4} = 14\sqrt[3]{1-\cdot 107}$$

$$= 14\sqrt[3]{\cdot 893} = 14 \times \cdot 9629 = \underline{13\cdot 48 \text{ inches.}}$$

Diameter of solid shaft = $\underline{13\cdot 48 \text{ inches}}$, or, say, $\underline{13\frac{1}{2} \text{ inches.}}$

(13) A mild steel shaft is required to transmit 200 horse power at a speed of 140 revolutions per minute. The shaft is to be 25 feet long, and must not spring more than 5 degrees under working conditions. Taking the modulus of rigidity of mild steel to be 12,000,000 lbs. per square inch, determine the diameter of the shaft.

We are concerned here with the stiffness of the shaft, and consequently use the relation,

$$T.M. = \frac{\phi G D^4}{584 l},$$

where T.M. = twisting moment applied, in pound-inches;
ϕ = maximum angle, in degrees, through which the shaft may be twisted;
G = modulus of rigidity, in lbs. per square inch;
D = diameter of shaft, in inches;
l = length ,, ,,

Now,

$$T.M. = \frac{63{,}000 \text{ H.P.}}{N},$$

where H.P.=horse power transmitted;
N=speed of shaft in revolutions per minute.

Substituting for H.P. and N their respective values,

$$\text{T.M.}=\frac{63{,}000\times 200}{140}.$$

Hence,
$$\frac{63{,}000\times 200}{140}=\frac{\phi GD^4}{584l}=\frac{5\times 12{,}000{,}000\times D^4}{584\times 25\times 12}$$

$$D^4=\frac{63{,}000\times 200\times 584\times 25\times 12}{140\times 5\times 12{,}000{,}000}=262{\cdot}8$$

$$D=\sqrt[4]{262{\cdot}8}=\underline{4{\cdot}02\text{ inches.}}$$

Required diameter of shaft=$\underline{4{\cdot}02\text{ inches,}}$ or, say, $\underline{4\text{ inches.}}$

135.—(14) Find the diameter of a shaft, which, when subject to a torsional stress of 6000 lbs. per square inch, will not spring more than 2 degrees in a length of 30 feet. The modulus of rigidity may be taken as 12,000,000 lbs. per square inch.

In this example, we are given the stress in the shaft, but not the twisting moment which produces that stress. We therefore use the formula which connects the stress with the angle of twist, viz.:—

$$\frac{f_s}{R}=\frac{\phi}{l}\times G,$$

where f_s=shear stress, in lbs. per square inch;
R=radius of shaft;
ϕ=angle of twist, in radians;
l=length of shaft, in inches;
G=modulus of rigidity.

Substituting the values given in the question,

$$\frac{6000}{R}=\frac{\frac{2}{57{\cdot}3}}{30\times 12}\times 12{,}000{,}000.$$

Notice that the angle is given in degrees. This must be converted into radians by dividing it by 57·3. Also, the length is given in feet, so that we must bring this to inches by multiplying by 12

Hence,
$$\frac{6000}{R}=\frac{{\cdot}0349\times 12{,}000{,}000}{30\times 12}.$$

$$R=\frac{30\times 12\times 6000}{{\cdot}0349\times 12{,}000{,}000}$$
$$=5{\cdot}157\text{ inches.}$$

$$\therefore \quad D=5{\cdot}157\times 2=\underline{10{\cdot}314\text{ inches.}}$$

The shaft would probably be made $\underline{10\frac{1}{2}\text{ inches}}$ diameter.

(15) Referring to the previous question, find the twisting moment that could be applied to the shaft.

The formula to be used is now,

$$\frac{\text{T.M.}}{\text{J}}=\frac{\text{C}\phi}{l},$$

where T.M. = twisting moment, in pound-inches;
J = moment of inertia of shaft about centre $=\frac{\pi d^4}{32}$ for a shaft of circular section;
C = modulus of rigidity;
ϕ = angle of twist, in radians;
l = length of shaft, in inches.

Substituting the values from the question, we have:—

$$\frac{\text{T.M.}}{\frac{\pi d^4}{32}}=\frac{12{,}000{,}000\times{\cdot}0349}{30\times12}$$

$$\frac{\text{T.M.}\times10{\cdot}2}{(10{\cdot}314)^4}=\frac{12{,}000{,}000\times{\cdot}0349}{30\times12},$$

and

$$\text{T.M.}=\frac{12{,}000{,}000\times{\cdot}0349\times(10{\cdot}314)^4}{30\times12\times10{\cdot}2}$$

$$=1{,}289{,}000 \text{ pound-inches}$$

$$=\underline{107{,}400 \text{ pound-feet.}} \quad \text{ANSWER.}$$

(16) Apply the Board of Trade formula to find the twisting moment that could be applied to the shaft of Example 14.

The formula referred to is,

$$\text{T}=\frac{140d^4a}{\text{L}},$$

where T = twisting moment, in pound-feet;
d = diameter of shaft, in inches;
a = angle of twist, in degrees;
L = length of shaft, in feet.

Taking the data from the previous example, and giving each letter its numerical value,

$$\text{T}=\frac{140\times(10{\cdot}314)^4\times2}{30}=\underline{105{,}500 \text{ pound-feet.}}$$

(17) Assuming the twisting moment found in the previous question, viz. 105,500 pound-feet, be applied to the shaft, find the stress in the material.

In order to solve this question, apply the formula,

$$\frac{\text{T.M.}}{\text{J}}=\frac{f_s}{\text{R}},$$

where T.M.=twisting moment, in pound-inches;
J=moment of inertia of shaft about centre$=\frac{\pi d^4}{32}$;
f_s=stress, in lbs. per square inch;
R=radius of shaft.

Putting in the value of J, and substituting $\frac{\text{D}}{2}$ for R,

$$\frac{\text{T}}{\frac{\pi d^4}{32}}=\frac{f_s}{\frac{\text{D}}{2}}.$$

From this,

$$f_s=\frac{\text{T}}{\frac{\pi d^3}{16}}=\frac{\text{T}\times 16}{\pi d^3}.$$

Substituting the known values,

$$f_s=\frac{105{,}500\times 12\times 16}{\pi\times(10\cdot314)^3}=5900 \text{ lbs. per square inch.}$$

$\therefore$ f_s=<u>5900 lbs. per square inch.</u>

This agrees with the value of the stress given in Question 14.

136.—(18) The piston of a steam engine is 40 inches diameter, and the effective pressure acting on it is 50 lbs. per square inch. Stroke 4 feet 6 inches. The engine crank shaft supports a flywheel which gives rise to a bending moment equal to ·785 of the twisting moment. Find the diameter of the crank shaft, assuming it to be solid.

The first thing to do is to find the maximum twisting moment, which may be taken to be equal to $p\times\frac{\pi}{4}\times d^2\times l$,

where p=effective pressure, in lbs. per square inch;
d=diameter of piston, in inches;
l=length of crank, in inches=one half the stroke.

$$\begin{aligned}\text{T.M.}&=p\times\frac{\pi}{4}\times d^2\times l\\&=50\times\cdot785\times 40\times 40\times\frac{4\cdot5}{2}\times 12\\&=\underline{1{,}696{,}000 \text{ pound-inches.}}\end{aligned}$$

$\therefore$ Twisting moment=<u>1,696,000 pound-inches.</u>

The bending moment is equal to ·785 of the twisting moment.

$$\text{B.M.} = \cdot785 \times 1{,}696{,}000$$
$$= \underline{1{,}331{,}000 \text{ pound-inches.}}$$
$$\therefore \text{ Bending moment} = \underline{1{,}331{,}000 \text{ pound-inches.}}$$

We have next to find the equivalent twisting moment. T.M._E. Thus,

$$\text{T.M.}_E = \text{B.M.} + \sqrt{\text{T.M.}^2 + \text{B.M.}^2},$$

where B.M. is the bending moment in pound-inches.

Substituting the known values of B.M. and T.M.,

$$\text{T.M.}_E = 1{,}331{,}000 + \sqrt{1{,}696{,}000^2 + 1{,}331{,}000^2}.$$

To obtain the numerical value of the quantity under the root sign, describe the triangle ABC (Fig. 67), making the side AB 1,696,000 units long, to some convenient scale, and the side BC 1,331,000 units long. Then the length of the side AC, to the same scale, gives us the value required, which is approximately 2,160,000. Hence,

$$\text{T.M.}_E = 1{,}331{,}000 + 2{,}160{,}000$$
$$= \underline{3{,}491{,}000 \text{ pound-inches.}}$$

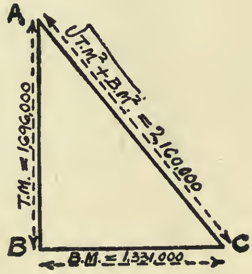

Fig. 67

This equivalent twisting moment will produce the same effect as the ordinary twisting moment and the bending moment together, and the next thing to do is to equate it to the torsional resistance of the shaft. Thus,

$$\text{T.M.}_E = \frac{\pi}{16} D^3 f_s,$$

where D = diameter of crank shaft;
f_s = allowable stress in shaft.

Rearranging,

$$D^3 = \frac{\text{T.M.}_E \times 16}{\pi \times f_s}$$

$$D = \sqrt[3]{\frac{\text{T.M.}_E \times 16}{\pi \times f_s}}.$$

Assume f_s to be 6500 lbs. per square inch.

Then, $$D = \sqrt[3]{\frac{3{,}491{,}000 \times 16}{\pi \times 6500}} = \sqrt[3]{2735} = 13\cdot98 \text{ inches.}$$

$\therefore$ Required diameter of shaft = $\underline{13\cdot98 \text{ inches}}$, or, say, $\underline{14 \text{ inches.}}$

(19) Two lengths of hollow shafting, 12 inches diameter externally, and 6 inches internally, are to be secured together by means of a solid flange coupling containing six bolts, the diameter of the bolt circle being 17 inches. Assuming the bolts to be made of the same material as the shaft, determine the necessary diameter of the bolts.

The required diameter is given by the relation,

$$d_b=\sqrt{\frac{D^4-d^4}{\cdot 3nr_cD}},$$

where d_b=diameter of bolts, in inches;
D=external diameter of shaft, in inches;
d=internal ,, ,, ,,
n=number of bolts;
r_c=radius of bolt centre circle.

In this example, $D=12$ inches, $d=6$ inches, $n=6$, $r_c=8{\cdot}5$ inches. Substituting these values in the formula,

$$d_b=\sqrt{\frac{12^4-6^4}{3\times6\times8{\cdot}5\times12}}=\sqrt{\frac{20{,}736-1296}{18\times102}}=\sqrt{10{\cdot}59}=\underline{3{\cdot}25\text{ inches.}}$$

$\therefore$ Diameter of bolts$=\underline{3\tfrac{1}{4}\text{ inches.}}$

Chapter IX

SHOCK OR IMPACT TESTS ON IRON AND STEEL

In addition to tensile compression and bending tests on iron and steel which have been described above, the student, on further acquaintance with the subject, will find many other forms of test in practical use. A description of these, some of which are only of limited application, is beyond the scope of the present work. Some reference, however, should be made to one of the most important, namely, the shock or impact test.

The tensile test has been referred to above as a means of determining whether steel or iron is brittle, and therefore unsuitable to withstand shocks. While this is true in general, it is not infallible in this respect. Many failures of engineering structures and engine parts have occurred clearly attributable to inherent brittleness in the material used, even in cases where under the tensile test the steel has shown a satisfactory ductility or percentage elongation. Recognition of this fact led a French engineer, M. Charles Frémont, to devise a method of testing in which a notched test piece of the steel is broken under a sudden blow.

Various types of machine have been constructed by different makers to carry out such tests, of which the Frémont, Charpy and Izod machines are the best known. They are, however, all alike in principle, and the following short description of the Frémont method as used for the past eighteen years by the well-known firm of steel makers, Messrs. Hadfields, Limited, of Sheffield, will illustrate the general principle.

The test piece A, in the form of a rectangular bar, notched centrally on its underside, rests on two supports, as shown in Fig. 68.

A falling weight, or tup B, fitted with a rounded knife edge C to concentrate the blow, falls on the test piece centrally, thus breaking it transversely through the notch. After breaking the test piece, the tup falls on to a copper cylinder D, which it compresses.

The amount of compression of the copper is a measure of the energy remaining in the tup after the fracture of the specimen. This amount of energy subtracted from the known energy of the tup before striking gives, therefore, the energy required to break the specimen.

The energy figure[1] thus obtained is a measure of the toughness of the material, a quality which is the converse of its brittleness. It is also useful to record the angle through which the specimen has been bent before fracture. This is done by fitting the two fragments together and measuring with a protractor.

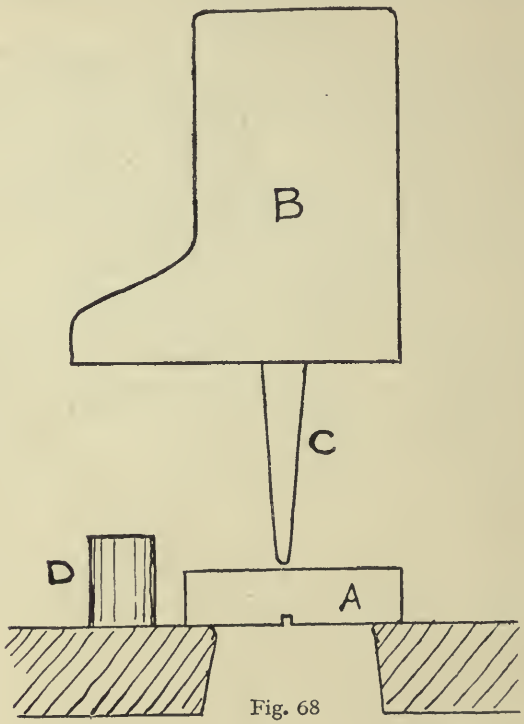

Fig. 68

The tup weighs 5 kilograms, and falls from a height of 4 metres; its striking energy is, therefore, 20 kilogram-metres (written KgM.).

In a particular test on a sample of mild steel, the copper cylinder measured 0·570 inch in length.

After compression by the tup, subsequent to fracture of the specimen, it measured 0·377 inch in length.

[1] The student will note that while the strength of a material in the tensile test is recorded in the form of a stress, that is, a force per unit area, in the case of the impact test energy is used, since not only the force of rupture is concerned, but also the extent of the deformation of the specimen, that is, the distance through which the force of rupture acts.

The amount of compression is, therefore (0·570—0·377) inch =0·193 inch in length.

Looking up the calibration curve for the copper cylinder, this corresponds to an energy of 16·7 KgM. expended on the copper.

The energy expended in breaking the testpiece is therefore 20·0—16·7=3·3 KgM.

As, also, the fragments fitted together showed an angle of bend of only 4 degrees, the material is far from satisfactory in its shock-resisting qualities. Good mild steel should give at least 10 KgM. in this test.

In a test of another specimen of mild steel the following were the data :—

Weight of tup, ...	5 Kg. (as before).
Height of drop, ...	4 metres ,, ,,
Striking energy of tup, . . .	20 KgM. ,, ,,
Length of copper cylinder before test,	0·572 inch.
Do. do. do. after test,	0·494 ,,
Compression of copper cylinder, . . .	0·078 inch.
Corresponding energy expended on copper (from calibration curve), . .	4·5 KgM.
Energy expended in breaking the test piece	(=20—4·5=15·5 KgM).
Angle of bend, ...	60 degrees.

The shock-resisting qualities of this specimen of mild steel are, therefore, represented by 15·5 KgM. × 60 degrees, which may be considered as satisfactory.

Results have shown this method of shock testing to indicate toughness or brittleness which is not rendered apparent by other forms of test. Shock or impact testing therefore forms a valuable adjunct to the tensile compression, bending, or other tests commonly used, and they are now in general use, being specified by certain Government Departments and other large users of steel.

An interesting example of the use of shock tests is in the case of mild steel commonly used for structural purposes such as ship plates, boiler plates, girders, etc. In the working up of such steel in large quantities it is occasionally subjected to very high temperatures for a longer period than is advisable. Under the tensile or other common forms of test no harmful results from this maltreatment are apparent, in fact, the steel shows quite a satisfactory tenacity and an excellent elongation under the tensile test.

On submitting the material to the Frémont shock test, it breaks in an absolutely brittle manner with the expenditure of a very slight amount of energy, and the fragments when fitted together show a bending angle of less than one degree. The fracture is coarsely crystalline. It is obvious that such a material, although satisfactory under the tensile test, is quite dangerous to use.

It is interesting to note that a similar notched specimen tested slowly will bend double without fracture.

Other examples might be quoted, particularly in the case of steels for aircraft, where the shock test has proved of undoubted service

Chapter X

CHAINS AND ROPES

CHAINS and ropes are in general subjected to tensile loads, and the determination of the stresses is a comparatively simple matter.

In the case of chains, apart from the tensile load, there is bending of the individual links, but it is not possible to calculate, with any degree of certainty, the stresses due to the bending action, and it is usual to determine the strength by considering merely the tensile stresses.

Chains are mostly made of tough fibrous wrought iron, because although this material is not so strong as mild steel, it is a more suitable material for welding, on which so much depends.

In the case of what are known as *weldless chains*, made in short lengths for slings, and cut out of a solid bar, mild steel may be employed with advantage, a mild steel chain of this form being probably twice as strong as one of wrought iron.

The ordinary chain is made in two principal forms, viz. the

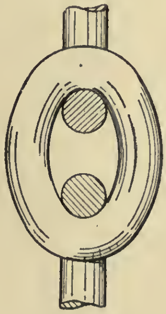

Fig. 69A

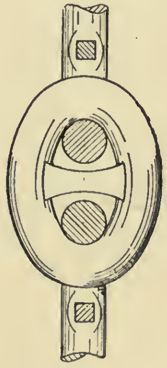

Fig. 69B

open link and the *stud link*. The former is shown in Fig. 69A, and the latter in Fig. 69B.

In the latter type, a short stay or stud, generally of cast iron, is placed across the shorter diameter of each link, its object being to prevent the sides of the link closing in as a result of the tension in the chain. The stud link chain is certainly stronger than the open link, some authorities regarding it as one and a half times as strong.

To find the stress in an ordinary chain, we must note that the two sides of each link resist the load.

Let D=diameter of the round bar from which the chain is made.
,, W=load on chain.
,, f_t=tensile stress in chain.

Then since $\text{Stress}=\dfrac{\text{Load}}{\text{Sectional area}}$,

$$f_t=\frac{W}{2\times\frac{\pi}{4}D^2},$$

i.e. $$f_t=\frac{2W}{\pi D^2},$$

and $$W=\frac{\pi D^2 f_t}{2}.$$

If we adopt a safe stress of 4 tons per square inch, then

$$W=\frac{\pi D^2\times 4}{2}=6{\cdot}28D^2.$$

Thus, Safe Load=$\underline{6{\cdot}28D^2}$ (for steady loads).

This simple formula may be used for ordinary open link chains which are not exposed to shock. The safe stress of 4 tons per square inch assumes a factor of safety of between four and five, which is satisfactory for steady loads.

Frequently, however, chains are very liable to rough usage, and in such cases a safe stress of not more than $2\frac{1}{2}$ tons per square inch should be adopted.

Our formula $$W=\frac{\pi D^2 f_t}{2}$$

then becomes $$W=\frac{\pi D^2\times 2{\cdot}5}{2},$$

i.e. $W=\underline{4D^2}$ (for unsteady loads).

Fatigue of Chains.—Perhaps no part of a structure or a machine is more liable to suffer from fatigue than a chain, particularly one used on a crane.

If accidents are to be avoided, therefore, it is very necessary that chains should be annealed from time to time; that is to say, they should be thoroughly and evenly heated, to a red heat, and then allowed to cool very slowly.

How often a chain should be annealed depends on circumstances. If the chain is not worked a great deal, and if it is not subjected to rough usage, once in twelve months may be sufficient, but in severe cases, as, for example, where chains are used in foundries for lifting molten metal, annealing every three months is advised.

Proof Stress.—This is a term the student should be familiar with. It is a term much used in connection with chains, and denotes the stress to which a chain is exposed when subjected to a test load or *proof load.*

The proof stress is generally about one half the ultimate stress of the material, and the working stress one half the proof stress for steady loads, but less of course where the chain is subjected to severe conditions of working.

The Admiralty specify a proof load of $12D^2$ for open link chains ($18D^2$ for stud link chains), which we see is roughly twice the working load as given by our formula,

$$\text{Safe Load} = 6{\cdot}28D^2.$$

Wire Ropes.—The strength of a wire rope or cable depends upon the number, size, and tensile strength of the wires of which it is composed. Except in the case of ropes for suspension bridges, where great flexibility is not required, hemp cores are frequently used in the construction of wire ropes, so that the strength of the rope cannot be determined simply from the diameter. These ropes are made of steel, often of very high tensile strength, over 100 tons per square inch in some cases, but it is not usual to adopt a lower factor of safety than six, especially for ropes which pass over pulleys, since the continual bending of the rope in passing round the pulleys severely strains the wires, and tends to cause fatigue and breakage of the strands, particularly if the pulleys are not of large diameter. In many cases a factor of safety of ten is adopted. The actual strengths of wire ropes are best obtained from the makers' lists.

Horizontally Suspended Chains and Ropes.—Examples of chains and ropes suspended horizontally, i.e. stretched across from one pillar or column to another, are seen in the case of the chains and ropes used for supporting suspension bridges, etc. These are loaded at various points along their length with loads which usually act vertically downwards, the sum total of the loads being of course that of the bridge.

A telegraph wire is a somewhat similar example, but of course has merely its own weight to support. Such wires have to be exposed to very considerable tension in order that they will not sag appreciably.

We shall now deduce an expression which will enable us to find the tension in a horizontally suspended chain or rope. Consider a chain or rope of a suspension bridge carrying a number of vertical loads. (See Fig. 70.)

The bending moment diagram for a beam carrying a uniformly distributed load is a parabola, so that the general shape of the chain or rope will be that of a parabola, since we may, without appreciable error, assume the chain to have a continuous curve, especially if the number of vertical suspension rods is great, as is generally the case in practice.

If the tops of the columns over which the chain passes are on the same level, then the lowest point of the chain will be immediately over the centre of the bridge platform, and a tangent to

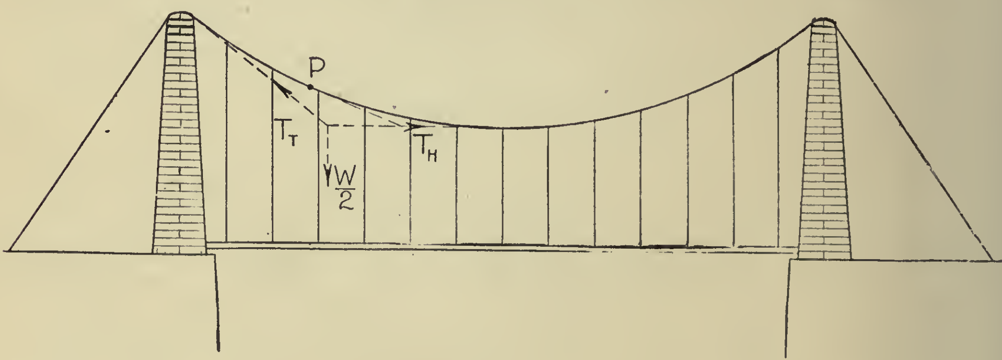

Fig. 70

the chain at this lowest point will be a horizontal line. A tangent to the chain at the top of one of the columns will meet the horizontal tangent at a point which is situated a horizontal distance of one quarter the span from the column.

Now consider the equilibrium of one half of the chain.

Let W=the total load on the chain.
T_H=the horizontal tension.
T_T=the tension at the top of the column.
L=the span.
D=the dip of the chain, i.e. the vertical distance from the top of the pillars to the level of the lowest point of the chain.

Now the half-chain is in equilibrium under the action of the three forces, viz. $\frac{W}{2}$, T_H, and T_T, and the lines of action of these three forces will of course meet in one and the same point.

Taking moments about the top of the column,

then, $$T_H \times D = \frac{W}{2} \times \frac{L}{4},$$

from which, $$T_H = \frac{WL}{8D}.$$

Thus, Horizontal tension = One-eighth load on chain $\times \frac{\text{span}}{\text{dip}}$.

To find the value of T_T, construct the triangle of forces abO (Fig. 70A), drawing ab vertical to represent the load $\frac{W}{2}$, bO parallel to the tangent to the chain at the lowest point, and Oa parallel to the tangent to the chain at the top of the column.

In the triangle abO, which is right-angled,

$$aO^2 = ab^2 + bO^2.$$

Now aO represents the tension T_T, ab the load on half the chain, viz. $\frac{W}{2}$, and bO the horizontal tension T_H.

$$\therefore \ T_T^2 = \left(\frac{W}{2}\right)^2 + T_H^2.$$

We have seen that T_H

$$= \frac{WL}{8D},$$

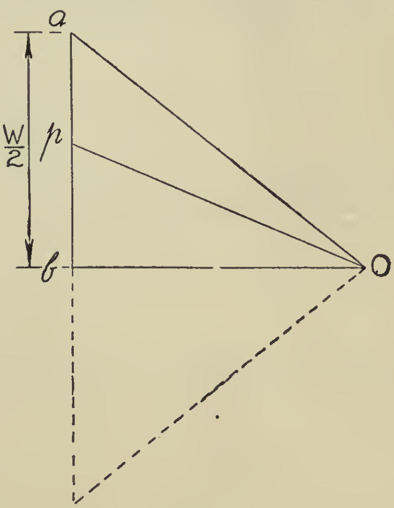

Fig. 70A

so that on substituting this value of T_H in the equation

$$T_T^2 = \left(\frac{W}{2}\right)^2 + T_H^2,$$

we obtain, $$T_T^2 = \frac{W^2}{4} + \left(\frac{WL}{8D}\right)^2$$

$$= \frac{W^2}{4} + \frac{W^2L^2}{64D^2}$$

$$= \frac{W^2}{4}\left(1 + \frac{L^2}{16D^2}\right).$$

$$\therefore \ T_T = \frac{W}{2}\sqrt{1 + \frac{L^2}{16D^2}}.$$

It will be seen from the triangle of forces that T_T is the greatest tension in the chain, and T_H the least.

To find the tension at any point P in the chain, draw a tangent to the chain at P, and from O in the triangle of forces draw the line O*p* parallel to this tangent.

Then in the right-angled triangle *pb*O,

$$pO^2 = pb^2 + bO^2.$$

Now $$\frac{pb}{ab} = \frac{l}{\frac{L}{2}},$$

where *l* is the horizontal length from a point directly under P to the centre of the chain.

Thus, $$pb = \frac{2ab \cdot l}{L}.$$

But *ab* represents one half the load, and $2 \cdot ab$ will therefore represent the full load.

Hence, $$pb = W \cdot \frac{l}{L}.$$

Therefore, since *p*O represents the tension at any point P, say T_P, and *b*O represents the horizontal tension, T_H, we have finally,

$$\underline{T_P = \sqrt{T_H^2 + \left(W \cdot \frac{l}{L}\right)^2}.}$$

If we apply this formula to find the greatest tension, i.e. the tension at the top of the supports, T_T, we find it reduces to the formula already given for T_T.

Thus, *l* is the horizontal length of chain from a point directly under the point being considered, which is at the top of the tower, to the centre of the chain ; obviously

$$l = \frac{L}{2}.$$

Hence, $$T_P = \sqrt{T_H^2 + \left(W \cdot \frac{\frac{L}{2}}{L}\right)^2}.$$

Now $$T_H = \frac{WL}{8D},$$

so that substituting this for T_H, we have

$$T_P=\sqrt{\left(\frac{WL}{8D}\right)^2+\left(\frac{W}{2}\right)^2}$$

$$=\sqrt{\frac{W^2L^2}{64D^2}+\frac{W^2}{4}}$$

$$=\sqrt{\frac{W^2}{4}\left(1+\frac{L^2}{16D^2}\right)}.$$

$$\therefore\quad T_P=\frac{W}{2}\sqrt{1+\frac{L^2}{16D^2}},$$

or, since the point chosen is that where the tension is greatest, i.e. at the top of the tower,

$$T_T=\frac{W}{2}\sqrt{1+\frac{L^2}{16D^2}}.$$

If we apply the formula which gives the tension at any point to find the tension at the centre of the chain, we note that the length of chain l is nil, so that

$$T_P=\sqrt{T_H{}^2+\left(W\cdot\frac{O}{L}\right)^2}$$

$$=\sqrt{T_H{}^2+O},$$

i.e. $$T_P=T_H.$$

Thus, the tension at the centre is equal to T_H, the minimum tension.

Length of Suspended Chain or Rope.—If a chain or rope be supposed to hang in the form of a parabolic curve, the length may be determined approximately from the formula,

$$L_C=L+\cdot 23D,$$

where L_C=length of chain or rope;
L=span, or horizontal distance between supports;
D=dip, or vertical distance from highest to lowest level of chain.

NOTE.—The actual curve which a chain or rope assumes, when not loaded like a suspension bridge chain or rope, is known as the *catenary*.

Worked Examples

(1) Find the loads which may be imposed on a $\frac{3}{8}$-inch, a $\frac{5}{8}$-inch, and a 1-inch chain, assuming (*a*) the chains are subjected to steady loads, (*b*) to unsteady loads.

What loads might be carried if the chains were of the stud link form instead of the open link?

(*a*) For steady loads,

$$W = 6{\cdot}28D^2,$$

where W = load, say in tons,

D = diameter of chain in inches.

For a $\frac{3}{8}$-inch chain,

$$W = 6{\cdot}28 \times {\cdot}375^2 = \underline{{\cdot}883 \text{ ton.}}$$

For a $\frac{5}{8}$-inch chain,

$$W = 6{\cdot}28 \times {\cdot}625^2 = \underline{2{\cdot}45 \text{ tons.}}$$

For a 1-inch chain,

$$W = 6{\cdot}28 \times 1^2 = \underline{6{\cdot}28 \text{ tons.}}$$

(*b*) For unsteady loads,

$$W = 4D^2.$$

For a $\frac{3}{8}$-inch chain,

$$W = 4 \times {\cdot}375^2 = \underline{{\cdot}562 \text{ ton.}}$$

For a $\frac{5}{8}$-inch chain,

$$W = 4 \times {\cdot}625^2 = \underline{1{\cdot}562 \text{ tons.}}$$

For a 1-inch chain,

$$W = 4 \times 1^2 = \underline{4 \text{ tons.}}$$

These results are for chains of the open link type.

If the chains were of the stud link type, the loads which might be carried would be $1\frac{1}{2}$ times those above determined.

(*a*) For steady loads (stud link chains),

$$W = {\cdot}883 \times 1{\cdot}5 = \underline{1{\cdot}325 \text{ tons}} \ (\tfrac{3}{8}\text{-inch chain}).$$

$$W = 2{\cdot}45 \times 1{\cdot}5 = \underline{3{\cdot}68 \text{ tons}} \ (\tfrac{5}{8}\text{-inch chain}).$$

$$W = 6{\cdot}28 \times 1{\cdot}5 = \underline{9{\cdot}42 \text{ tons}} \ (1\text{-inch chain}).$$

(*b*) For unsteady loads,

$$W = \cdot 562 \times 1\cdot 5 = \underline{\cdot 843 \text{ ton}} \ (\tfrac{3}{8}\text{-inch chain}).$$

$$W = 1\cdot 562 \times 1\cdot 5 = \underline{2\cdot 34 \text{ tons}} \ (\tfrac{5}{8}\text{-inch chain}).$$

$$W = 4 \times 1\cdot 5 = \underline{6 \text{ tons}} \ (1\text{-inch chain}).$$

(2) A steel wire rope is stretched between two supports in the same horizontal line 90 feet apart, and the dip of the rope is 9 feet. This wire rope sustains a load which may be estimated as equal to 4 cwts. per horizontal foot of span. Determine the greatest and the least tensions produced in the wire rope by this load. (B. of E. Mech. Exam.).

The greatest tension is given by the formula,

$$T_T = \frac{W}{2}\sqrt{1 + \frac{L^2}{16D^2}},$$

where T_T = tension at top of supports,

W = total load on rope;

L = span in feet;

D = dip of rope in feet.

Now $W = 90 \times 4 = 360$ cwts., $L = 90$ feet, and $D = 9$ feet.

Substituting these values,

$$T_T = \frac{360}{2}\sqrt{1 + \frac{90^2}{16 \times 9^2}} = 485 \text{ cwts.}$$

$\therefore$ Greatest tension = $\underline{485 \text{ cwts.}}$

The least tension is given by the formula,

$$T_H = \frac{WL}{8D}.$$

$$\therefore \ T_H = \frac{360 \times 90}{8 \times 9} = \underline{450 \text{ cwts.}}$$

$\therefore$ Least tension = $\underline{450 \text{ cwts.}}$

$$\left.\begin{array}{r} \text{Greatest tension} = \underline{485 \text{ cwts.}} \\ \text{Least tension} = \underline{450 \text{ cwts.}} \end{array}\right\} \ \text{ANSWERS.}$$

(3) A foot bridge 10 feet in width is carried over a river 100 feet in width by two cables of uniform section, with a dip of 10 feet at the centre. Find the greatest pull on the cables, their

cross-sectional area, length, and weight from the following data: Maximum load on platform, 120 lbs. per square foot; working stress in metal of cables, 4 tons per square inch; weight of cable material, 484 lbs. per cubic foot. (A.M.I.C.E. Exam.)

Superficial area of bridge $=100\times 10=1000$ square feet.

Load per square foot $=120$ lbs.

$\therefore$ Total load $=1000\times 120=120{,}000$ lbs.

$=53{\cdot}6$ tons.

Load from bridge to be carried by each cable is therefore

$53{\cdot}6\div 2=26{\cdot}8$ tons.

In addition to this, each cable has to carry its own weight. We do not, however, know the weight of the cables, nor can we get it at once since we do not know their size.

In many cases, in order to simplify the problem, the weight of the cables would be neglected, since compared with the load carried it is usually small.

If, however, it is not desired to ignore the weight of the cables, they may be assumed a certain size, and the weight calculated accordingly. Providing one shows good judgment in assuming the sizes, the results will then be quite accurate enough for all practical purposes.

The exact determination is a little more troublesome, but we shall here adopt the scientific method.

Let d = diameter of cables in inches,

L_C = length of cables in feet,

then, Volume of one cable $=\dfrac{\frac{\pi}{4}d^2}{144}\times L_C$ (cubic feet).

Weight of cable $=\dfrac{\frac{\pi}{4}d^2}{144}\times L_C\times w$,

where w = weight of one cubic foot of the material.

Now $L_C=L+{\cdot}23D$,

where L = span in feet,

D = dip in feet.

Since the span is stated to be 100 feet, and the dip 10 feet,

$L_C=100+{\cdot}23\times 10=102{\cdot}3$ feet.

Now the weight of a cubic foot of material is given as 484 lbs., i.e. ·216 ton.

$$\therefore\ \text{Weight of cable}=\frac{\frac{\pi}{4}d^2}{144}\times 102{\cdot}3\times{\cdot}216$$

$$={\cdot}1205d^2\ \text{tons.}$$

Hence, Total load supported by one cable

$$=26{\cdot}8+{\cdot}1205d^2\ (\text{tons}).$$

The formula which gives the maximum tension in each cable is

$$T_T=\frac{W}{2}\sqrt{1+\frac{L^2}{16D^2}},$$

where W=the total load supported by one cable.

Substituting for W, the value $26{\cdot}8+{\cdot}1205d^2$, and for L and D, 100 and 10 respectively,

$$T_T=\left(\frac{26{\cdot}8+{\cdot}1205d^2}{2}\right)\sqrt{1+\frac{100^2}{16\times 10^2}}$$

$$=(13{\cdot}4+{\cdot}06025d^2)\sqrt{1+6{\cdot}25}$$

$$=(13{\cdot}4+{\cdot}06025d^2)2{\cdot}69.$$

Hence, Maximum tension$=(13{\cdot}4+{\cdot}06025d^2)2{\cdot}69$ tons.

Since d represents the cable diameter, the sectional area is $\frac{\pi}{4}d^2$.

Therefore, $$\text{Stress}=\frac{\text{Tension}}{\text{Sectional area}}$$

$$=\frac{(13{\cdot}4+{\cdot}06025d^2)2{\cdot}69}{\frac{\pi}{4}d^2}.$$

We are told that the stress is to be 4 tons per square inch.

$$\therefore\ 4=\frac{(13{\cdot}4+{\cdot}06025d^2)2{\cdot}69}{\frac{\pi}{4}d^2}.$$

Thus, $$\pi d^2=36+{\cdot}162d^2,$$

$$3{\cdot}1416d^2-{\cdot}162d^2=36,$$

$$\therefore\ d=\sqrt{\frac{36}{2{\cdot}9796}}=\underline{3{\cdot}48}.$$

Hence, Diameter of cable $=3{\cdot}48$ inches.

$$\text{Sectional area} = \frac{\pi}{4} \times 3{\cdot}48^2 = \underline{9{\cdot}5 \text{ square inches.}}$$

$$\text{Weight of cable} = \frac{9{\cdot}5}{144} \times 102{\cdot}3 \times {\cdot}216$$

$$= \underline{1{\cdot}46 \text{ tons.}}$$

We have seen that the maximum tension is equal to

$$(13{\cdot}4 + {\cdot}06025 d^2) 2{\cdot}69 \text{ tons.}$$

Substituting for d the value $3{\cdot}48$ inches, we have

$$\text{Maximum tension} = (13{\cdot}4 + {\cdot}06025 \times 3{\cdot}48^2) 2{\cdot}69$$

$$= \underline{38 \text{ tons.}}$$

The answers to the problem are therefore as follow :—

$$\left.\begin{array}{r} \text{Greatest pull on cables} = \underline{38 \text{ tons.}} \\ \text{Sectional area of cables} = \underline{9{\cdot}5 \text{ square inches}} \text{ (each).} \\ \text{Length of cables} = \underline{102{\cdot}3 \text{ feet.}} \\ \text{Weight of cables} = \underline{1{\cdot}46 \text{ tons}} \text{ (each).} \end{array}\right\} \text{ANSWERS.}$$

Chapter XI

REVOLVING RING

THE determination of the stresses in a ring revolving about its centre is simple, but nevertheless important, since it has an important application in practice in regard to revolving wheels and pulleys.

Our problem is very similar, as we shall presently see, to finding the stress in a thin cylindrical vessel exposed to internal pressure.

Consider the ring shown in Fig. 71, and suppose this to be rotating at a uniform speed about its centre.

Fig. 71

Now according to Newton's First Law of Motion : "Every body continues in its state of rest, or of uniform motion in a straight line, unless compelled by impressed forces to change that state."

Since all the particles which make up the ring move in a circle, instead of a straight line, it follows that every particle must be continuously acted upon by some force whose line of action is towards the centre of the ring. This force is termed the *centripetal force.*

According to Newton's Third Law of Motion : "To every action there is an equal and opposite reaction." Hence, there must be a force equal to the centripetal force, but acting in the opposite direction, and this equal and opposite force is termed the *centrifugal force.*

Fig. 72

Now the summation of all these small centrifugal forces acting in the ring, i.e. the total centrifugal force, is tending to burst the ring into two equal halves. Fig. 72 shows one half of the ring, the arrows representing the centrifugal forces acting radially all round the ring. The case is seen to be analogous with that of a thin cylindrical shell under internal pressure. The total force acting on one half of the ring will be balanced by the total stress acting at the sections at A and B.

In the case of the thin shell, if p is the internal pressure in lbs. per square inch, d the diameter of the vessel in inches, t the thickness of the plates in inches, and f_t the tensile stress in the metal, in lbs. per square inch, then we saw in Chapter V. that

$$pd = 2f_t t.$$

In the case of the ring, instead of the pressure p, we have centrifugal force per unit length of the ring.

Now it is a simple matter to prove that the centrifugal force in a rotating body is

$$F = \frac{Wv^2}{gr},$$

where F = the centrifugal force, say in lbs.;
W = the weight of the body in lbs.;
v = the linear velocity in feet per second;
r = the radius of gyration in feet;
g = acceleration due to gravity, viz. 32·2 feet per second per second.

It will be convenient to consider a length of ring of one foot (because the linear velocity of the ring is in feet per second), and a sectional area of one square inch.

If W_1 be the weight in lbs. of this portion of the rim,

then $$F = \frac{W_1 v^2}{gr}.$$

This is the pressure on each unit of length of the ring, due to the centrifugal forces, and it corresponds with p in the equation,

$$pd = 2f_t t.$$

Since $d = 2r$, where r = the radius of the shell, then

$$p \times 2r = 2f_t t,$$

or $$pr = f_t t.$$

If t is taken to be 1 inch, then

$$pr = f_t.$$

If now we substitute for p the value $\frac{W_1 v^2}{gr}$, we obtain

$$f_t = \frac{W_1 v^2}{gr} \times r = \frac{W_1 v^2}{g}.$$

If we imagine the ring to be made of cast iron, then the value of W_1 is $\cdot 26 \times 1 \times 12 = 3\cdot 12$ lbs., a cubic inch of cast iron weighing $\cdot 26$ lb.

Hence
$$f_t = \frac{W_1 v^2}{g}$$
$$= \frac{3\cdot 12 v^2}{32\cdot 2} = \frac{v^2}{10} \text{ (approx.)}.$$

Since the weight of wrought iron or steel is about ·28 lb. per cubic inch, the same result applies approximately for wrought iron or steel rings.

We see from this that the stress in the ring due to centrifugal force depends only on the linear velocity, and is independent of the radius or the diameter of the ring, or of the section.

Thus, the stress in a rotating pulley or flywheel (if we neglect the effect of the arms) due to centrifugal action, is quite independent of the sectional area of the rim.

In most cases in engineering practice, if we find a part is highly stressed, we increase the section, but in the case of a pulley or flywheel this is useless, as it would not reduce the stress, because the latter is governed entirely by the velocity of the rim, or the "rim speed," as it is called. It is necessary to observe another important fact.

The stress varies as the square of the velocity. This means that if the velocity is doubled the stress is increased fourfold, whilst if the velocity is trebled the stress is increased ninefold.

We can readily understand from this why, if an engine commences to race, due, say, to failure of the governor gear, considerable risk of the flywheel bursting is involved, since at double the speed the stresses in the rim are increased fourfold, and at this speed, if the factor of safety is only four, the stress will have reached the breaking stress.

Rotating pulleys and wheels must therefore not be allowed to run beyond a certain speed, which will be governed by the material.

Most pulleys and wheels are made of cast iron, and the limiting speed for this is commonly taken to be a mile per minute, i.e. 88 feet per second.

Of late years there has been a tendency to introduce wrought iron and steel into the construction of large engine flywheels. One of the safest and best types of wheel is that in which the rim is made channel-shaped, and is wound round with many turns of steel wire of high tensile strength. Such a wheel may be safely run at speeds three times as great as those which would be safe for cast iron wheels.

In connection with our reference to pulleys and wheels, it is necessary to remark that the stresses in the rim are determined correctly by the formula given, providing we neglect the effect of the arms; in other words, if we assume the pulley or wheel to be a plain ring.

The effect of the arms is to cause bending stresses to be introduced, because each arm constrains the rim at one point, the result being that, as the forces acting on the wheel tend to make each portion of the rim move outwards, each length of rim between any two consecutive arms is in reality a beam fixed at the ends, and loaded uniformly along its length. (See Fig. 73.)

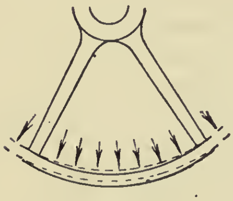

Fig. 73

Where small wheels are concerned, these bending stresses are not usually important, because the distance between the arms, which corresponds to the span of the beam, is small; but in the case of large wheels, the stresses due to bending may be quite as serious as those due to centrifugal tension.

Worked Examples

(1) A pulley rim is 4 feet diameter, and the speed of revolution is 160 per minute. Find the stress in the rim. What would be the stress if the speed were doubled?

Let f_t = stress in lbs. per square inch.

Then $$f_t=\frac{v^2}{10},$$

where v = linear velocity of rim in feet per second.

Now $$v=\frac{4\times\pi\times 160}{60}=33\cdot 5 \text{ feet per second.}$$

$$\therefore\ f_t=\frac{33\cdot 5^2}{10}=\underline{112\cdot 2 \text{ lbs. per square inch.}}$$

Stress in rim = <u>112·2 lbs. per square inch.</u>

If the speed were doubled, the stress would be increased fourfold, since it varies as the square of the rim speed.

Stress in rim = 112·2 × 4 = <u>448·8 lbs. per square inch.</u>

Stress in rim (at 160 revs.) = <u>112·2 lbs. per square inch.</u>
" " (" 320 ") = <u>448·8 lbs. per square inch.</u> } Answers.

(2) An engine flywheel is 20 feet diameter, and runs at a speed of 60 revolutions per minute. Find the stress in the rim due to centrifugal action, neglecting the effect of the arms. Find also the greatest speed at which the wheel may be safely run if the stress is not to exceed 1000 lbs. per square inch.

$$\text{Stress}=\frac{v^2}{10},$$

where v=rim speed in feet per second.

Now $$v=\frac{20\times\pi\times 60}{60}=20\pi \text{ feet per second,}$$

$$\therefore\ \text{Stress}=\frac{(20\pi)^2}{10}=\underline{394{\cdot}8 \text{ lbs. per square inch.}}$$

To find the greatest speed at which the wheel may be safely run, first find the greatest permissible value of v. Call this v_1.

Thus, $$f_t=\frac{{v_1}^2}{10}.$$

Since f_t is limited to 1000 lbs. per square inch,

$$1000=\frac{{v_1}^2}{10},$$

$$\therefore\ v_1=\sqrt{10{,}000}=\underline{100 \text{ feet per second.}}$$

Let N_1=the maximum speed in revolutions per minute.

Now $$v_1=\frac{\pi D N_1}{60},$$

$$\therefore\ N_1=\frac{60\times v_1}{\pi\times D}$$

$$=\frac{60\times 100}{\pi\times 20}$$

$$=\underline{95{\cdot}5 \text{ revolutions per minute.}}$$

$$\left.\begin{array}{r}\text{Stress in rim}=\underline{394{\cdot}8 \text{ lbs. per square inch.}}\\ \text{Maximum speed of revolution}\\ =\underline{95{\cdot}5 \text{ revolutions per minute.}}\end{array}\right\}\text{ANSWERS.}$$

Chapter XII

SQUARE SHAFTS

The great majority of shafts used in practice are of round section. It is indeed the exception to find any other than the round section employed. Occasionally, however, shafts of square section are employed (in connection with travelling cranes, for example), and on this account, and further, because springs of square section are much used in practice, and the consideration of the strength of springs is based on a knowledge of the strength of shafts, we shall consider here very briefly the strength of shafts of square section.

In a round shaft, the shearing stress is uniform at all points equidistant from the axis, and the stress at any point varies as the distance of the point from the axis. This does not apply to shafts of any other section than the round, and, in consequence, the determination of the strength of non-circular shafts is a difficult problem. The problem, however, was investigated as far back as 1856 by St. Venant, who found that in the case of a square shaft the maximum intensity of stress occurs at the middle of each side of the square, and the torsional resistance is equal to $\cdot 208 S^3 f_s$, where S is the length of a side of the square, and f_s the shear stress.

Hence, if T.M. = the twisting moment,

then $$f_s = \frac{\text{T.M.}}{\cdot 208 S^3},$$

and $$\underline{\text{T.M.} = \cdot 208 S^3 f_s.}$$

This formula enables us to find the strength of a square shaft.

The corresponding formula for a solid round shaft of diameter D is

$$\text{T.M.} = \frac{\pi}{16} D^3 f_s.$$

As regards the stiffness of a square shaft, St. Venant found that the torsional rigidity of a square shaft was $\cdot 84$GJ, as against GJ in the case of a round shaft. (See top of p. 233.)

In dealing with the stiffness of round shafts, Chapter VIII., we deduced the formula,

$$\frac{\text{T.M.}}{\text{J}} = \frac{\varphi G}{l},$$

where $\quad$ T.M. = twisting moment;
J = moment of inertia of shaft;
φ = angle of twist in radians;
l = length of shaft;
G = modulus of rigidity of material;

and from this we see that for a round shaft,

$$\varphi=\frac{\text{T.M.}\times l}{\text{JG}}.$$

Hence, for the square shaft,

$$\varphi=\frac{\text{T.M.}\times l}{\cdot 84\text{JG}}.$$

For a round shaft, $\quad J=\frac{\pi D^4}{32}$,

so that $\quad \varphi=\frac{\text{T.M.}\times l}{\frac{\pi D^4}{32}\times G}=\frac{32\text{T.M.}l}{\pi G D^4}$,

the formula which we obtained for the angle φ (in radians) on p. 184.

Multiplying by 57·3, to obtain the angle φ in degrees, we obtain

$$\varphi\ (\text{degrees})=\frac{584\text{T.M.}l}{GD^4}\ (\text{round shaft}),$$

which is the general formula we used in dealing with the stiffness of a round shaft.

Our corresponding formula for the square shaft is obtained by substituting for J in the formula,

$$\varphi=\frac{\text{T.M.}\times l}{\cdot 84\text{JG}},$$

the value $\frac{S^4}{6}$, (this being the moment of inertia of a square shaft), where S is the length of the side of the square.

Hence, $\quad \varphi\ (\text{radians})=\frac{\text{T.M.}\times l}{\cdot 84\frac{S^4}{6}G}=\frac{6\text{T.M.}l}{\cdot 84S^4G}.$

Multiplying by 57·3, to obtain the angle φ in degrees, we obtain

$$\varphi\ (\text{degrees})=\frac{410\text{T.M.}l}{GS^4}\ (\text{square shaft}),$$

and $\quad \text{T.M.}=\frac{\varphi G S^4}{410l}.$

WORKED EXAMPLES

(1) Find the size of a mild steel square shaft to transmit 80 horse power at a speed of 100 revolutions per minute. Allowable shear stress, 7000 lbs. per square inch.

This question is the same as Question (3) in the Worked Examples of Chapter VIII., p. 198, but we are dealing with a square instead of a round shaft. We might anticipate that the size of the shaft, as measured by the length of a side, will be a little less than the size of the round shaft, as measured by its diameter.

We find the size of a square shaft from the relation,

$$\text{T.M.} = \cdot 208 S^3 f_s,$$

where T.M. = twisting moment, say in pound-inches;
S = length of side of square of shaft;
f_s = allowable shear stress, in lbs. per square inch.

Now
$$\text{T.M.} = \frac{63{,}000 \text{ H.P.}}{N},$$

where H.P. = the horse power to be transmitted,

and N = the shaft speed in revolutions per minute.

$$\therefore \text{T.M.} = \frac{63{,}000 \times 80}{100} = 50{,}400 \text{ pound-inches.}$$

Equating this to the torsional resistance,

$$50{,}400 = \cdot 208 S^3 f_s.$$

Substituting for f_s the value 7000 lbs. per square inch, we have finally,

$$S^3 = \frac{50{,}400}{\cdot 208 \times 7000} = 34 \cdot 6,$$

$$\therefore S = \sqrt[3]{34 \cdot 6} = \underline{3\tfrac{1}{4} \text{ inches}} \text{ (approx.)}.$$

Thus, the shaft is seen to be slightly less than the round shaft, which we found to be $3\frac{3}{8}$ inches diameter. The round shaft is more economical in material, the relative sectional areas or weights being as

$$\frac{\pi}{4}d^2 : S^2 = \cdot 785 \times 3 \cdot 375^2 : 3 \cdot 25^2$$
$$= 8 \cdot 95 \cdot : 10 \cdot 55$$
$$= 1 : 1 \cdot 18.$$

Required size of shaft = $\underline{3\frac{1}{4} \text{ inches (square)}}$. ANSWER.

(2) Find the relative strengths and weights of a 3-inch round shaft, a 3-inch square shaft, and a rectangular shaft, $3\frac{1}{2}$ inches by $2\frac{1}{2}$ inches.

The torsional strengths are $\frac{\pi}{16}D^3 f_s$ for the round shaft, $\cdot 208 S^3 f_s$ for the square shaft, and $\cdot 294 \frac{B^2H^2}{\sqrt{B^2+H^2}} f_s$ for the rectangular shaft, D representing the diameter of the round shaft, S the length of the side of the square shaft, B the breadth of one side, and H that of the other for the rectangular shaft, f_s representing the shear stress in each case.

Hence the relative strengths are as

$$\frac{\pi}{16}D^3 : \cdot 208 S^3 : \cdot 294 \frac{B^2H^2}{\sqrt{B^2+H^2}}$$

Now D=3 inches, S=3 inches, B=$3\frac{1}{2}$ inches, and H=$2\frac{1}{2}$ inches therefore the relative strengths are as

$$\frac{\pi}{16} \times 3^3 : \quad \cdot 208 \times 3^3 : \quad \cdot 294 \frac{3{\cdot}5^2 \times 2{\cdot}5^2}{\sqrt{3{\cdot}5^2+2{\cdot}5^2}}$$

$$5{\cdot}3 : 5{\cdot}62 \qquad : 5{\cdot}23$$

$$\underline{1} : \underline{1{\cdot}06} \qquad : \underline{\cdot 987}$$

(*Round*) (*Square*) (*Rectangular*)

The relative weights are as the relative sectional areas, viz. as

$$\frac{\pi}{4}D^2 \quad : \quad S^2 \quad : \quad BH,$$

$$\cdot 785 \times 3^2 \quad : \quad 3^2 \quad : \quad 3{\cdot}5 \times 2{\cdot}5$$

$$7{\cdot}07 \quad : \quad 9 \quad : \quad 8{\cdot}75$$

$$\underline{1} \quad : \underline{1{\cdot}273} : \quad \underline{1{\cdot}24}$$

(*Round*) (*Square*) (*Rectangular*)

	Round.	*Square.*	*Rectangular.*	
Relative Strengths,	1	1·06	·987	ANSWERS.
,, Weights,	1	1·273	1·24	

(3) A square shaft is 3·7-inch side, and 25 feet long. It transmits 200 horse power when running at a speed of 140 revolutions per minute. Find the angle of twist in degrees.

The formula for a square shaft is

$$\text{T.M.} = \frac{\varphi G S^4}{410 l},$$

where T.M. = twisting moment in pound-inches;
φ = angle of twist in degrees;
S = length of side of square of shaft in inches;
l = length of shaft in inches;
G = modulus of rigidity = 12,000,000 lbs. per square inch.

We require the angle φ.

Re-arranging, $\varphi = \frac{\text{T.M.} \times l \times 410}{GS^4}$.

Now $\text{T.M.} = \frac{63{,}000 \text{ H.P.}}{N}$,

where H.P. = horse power transmitted,

and N = speed in revolutions per minute.

The horse power is stated to be 200, and the speed 140 revolutions per minute.

$$\therefore \text{ T.M.} = \frac{63{,}000 \times 200}{140}.$$

Now $l = 25 \times 12$ inches, G = 12,000,000 lbs. per square inch, and S = 3·7 inches.

Substituting in the expression for φ, we have

$$\varphi = \frac{63{,}000 \times 200 \times 25 \times 12 \times 410}{140 \times 12{,}000{,}000 \times 3{\cdot}7^4}$$

$$= \underline{4{\cdot}92 \text{ degrees.}}$$

$\therefore$ Angle of twist = $\underline{4{\cdot}92 \text{ degrees.}}$ ANSWER.

Chapter XIII

SPRINGS

Helical Springs.—A helical spring may be regarded as a thick wire or a very small shaft coiled into a helix. In practice, the wire or shaft is generally either of round or square section.

When a spring of this form is subjected to a compressive or a tensile load, it tends to fail mainly by torsion. If the slope of the helix is not small, in other words, if the obliquity of the coils is considerable, there is also a certain amount of bending, which, strictly speaking, should be taken into account.

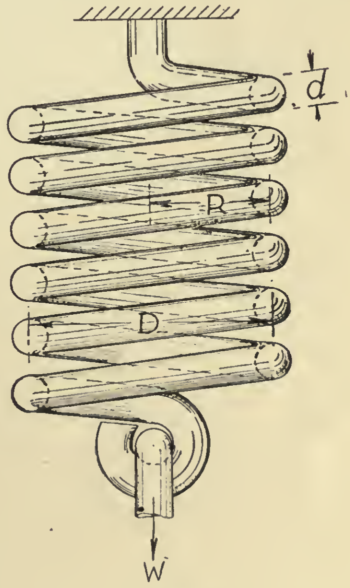

Fig. 74

In most cases in practice, however, the slope of the helix is so small that the question of bending may be ignored, and in what follows we shall assume that the stresses imposed on the coils are purely torsional.

We have stated that a helical spring may be regarded as a very small shaft coiled into a helix, and our knowledge of the strength of shafts will help us in dealing with the strength of helical springs.

Referring to Fig. 74, we see that the effect of a load acting in the direction of the axis of the spring will be to exert a twisting moment, the value of which will be WR, where W is the load and R the radius of the spring; i.e. the distance from the axis to the centre of the coils.

Now in Chapter VIII., dealing with the "Strength of Shafts," we deduced important relations connecting the twisting moment, the moment of inertia, the stress in the material, and the angle of twist of a shaft, etc.

The relations referred to are,

$$\frac{\text{T.M.}}{\text{J}}=\frac{\varphi G}{l}=\frac{f_s}{R} \text{ (see page 186)},$$

where T.M. = twisting moment in pound-inches;
J = moment of inertia of shaft;
φ = angle of twist;
G = modulus of rigidity of the shaft material;
l = length of shaft;
f_s = shear stress in shaft;
R = radius of cross-section of shaft.

We shall make use of this relation in finding the strength of a spring. As we are now going to let R represent the radius of the spring, it will be convenient to substitute R in the formula just stated by r, which will denote the radius of the section of the spring.

Since $$\frac{\text{T.M.}}{\text{J}}=\frac{f_s}{r},$$

then $$\text{T.M.}=\frac{f_s\text{J}}{r}.$$

Now in the case of the spring, we have seen that

$$\text{T.M.}=\text{WR}.$$

Substituting this for T.M., we have,

$$\text{WR}=\frac{f_s\text{J}}{r}.$$

For a round shaft or wire

$$\text{J}=\frac{\pi d^4}{32},$$

where d = diameter of shaft or wire.

Substituting this value for J, and $\frac{d}{2}$ for r,

$$\text{WR}=\frac{f_s\pi d^4}{32\frac{d}{2}}$$

$$=\frac{\pi f_s d^3}{16}.$$

If D = diameter of spring, then since $\text{R}=\frac{\text{D}}{2}$, we have

$$\text{W}\times\frac{\text{D}}{2}=\frac{\pi f_s d^3}{16}.$$

Hence, $$f_s=\frac{16\text{WD}}{2\pi d^3}=\frac{2{\cdot}55\text{WD}}{d^3}.$$

Hence, for a round coil helical spring,

$$\left.\begin{aligned} f_s &= \frac{2{\cdot}55\text{WD}}{d^3} \\ \text{and} \qquad \text{W} &= \frac{f_s d^3}{2{\cdot}55\text{D}} \end{aligned}\right\} \text{(for round coil helical springs).}$$

As regards springs of square section, we have seen that for a square shaft (see previous Chapter)

$$\text{T.M.} = {\cdot}208\text{S}^3 f_s,$$

where S = length of side of square.

Hence for the spring we have,

$$\text{W} \times \frac{\text{D}}{2} = {\cdot}208\text{S}^3 f_s,$$

and

$$f_s = \frac{\text{WD}}{{\cdot}416\text{S}^3} = \frac{2{\cdot}4\text{WD}}{\text{S}^3}.$$

Therefore, for a helical spring with coils of square section,

$$\left.\begin{aligned} f_s &= \frac{2{\cdot}4\text{WD}}{\text{S}^3} \\ \text{and} \qquad \text{W} &= \frac{f_s \text{S}^3}{2{\cdot}4\text{D}} \end{aligned}\right\} \text{(for square coil helical springs).}$$

The foregoing formulæ show the relation between the load on a spring and the stress.

We shall next find an expression for the amount of elongation or shortening of a round helical spring for a given load.

Referring again to the original formula,

$$\frac{\text{T.M.}}{\text{J}} = \frac{\varphi \text{G}}{l},$$

we note that

$$\varphi = \frac{\text{T.M.} \times l}{\text{J} \times \text{G}}.$$

Now the amount of extension or compression of the spring will be equal to the product of the angle of twist and the radius R of the spring. Call the extension or the compression δ.

Then

$$\delta = \varphi \times \text{R} = \varphi \times \frac{\text{D}}{2}.$$

Substituting for φ, the value $\dfrac{\text{T.M.} \times l}{\text{J} \times \text{G}}$,

$$\delta = \frac{\text{T.M.} \times l \times \frac{\text{D}}{2}}{\text{J} \times \text{G}}.$$

Since the twisting moment T.M. is equal to $W\times\frac{D}{2}$, and the total length of the spring is equal to $\pi\times D\times N$ (very nearly), where N is the number of the coils, then

$$\delta=\frac{W\times\frac{D}{2}\times\pi DN\times\frac{D}{2}}{J\times G}.$$

Now $$J=\frac{\pi d^4}{32},$$

and $$\therefore\quad \delta=\frac{W\times\frac{D}{2}\times\pi DN\times\frac{D}{2}}{\frac{\pi d^4}{32}G}$$

$$=\frac{8WD^3N}{Gd^4}.$$

$$\therefore\quad \underline{\delta=\frac{8WD^3N}{Gd^4}}\ \text{(for round coil helical springs)}.$$

This expression enables us to find the amount of extension or compression of a round helical spring caused by a load W.

Springs in general are made of steel, the value of G for which is usually taken as 12,000,000 lbs. per square inch.

Hence, for steel springs,

$$\delta=\frac{8WD^3N}{12,000,000d^4},$$

i.e. $$\underline{\delta=\frac{WD^3N}{1,500,000d^4}}\ \text{(for round coil helical springs)}.$$

For springs with coils of square section a formula may be derived in a similar manner.

Since the torsional rigidity for a square shaft is ·84JG instead of JG, we have

$$\delta=\frac{T.M.\times l\times\frac{D}{2}}{\cdot 84J\times G}.$$

Substituting $W\times\frac{D}{2}$ for T.M. and πDN for l,

$$\delta=\frac{W\times\frac{D}{2}\times\pi DN\times\frac{D}{2}}{\cdot 84J\times G}.$$

Now J for a square shaft$=\frac{S^4}{6}$,

where S=length of one side of the square.

Hence,
$$\delta=\frac{W\times\frac{D}{2}\times\pi DN\times\frac{D}{2}}{\cdot84\times\frac{S^4}{6}\times G}$$

$$=\frac{5\cdot62WD^3N}{GS^4}\text{ (for square coil helical springs).}$$

Taking the value of G as 12,000,000, we have finally,

$$\delta=\frac{WD^3N}{2,135,000S^4}\text{ (for square coil helical springs).}$$

Worked Examples

(1) A closely coiled cylindrical spring, 6 inches in mean diameter, is made out of $\frac{3}{8}$-inch steel wire. What direct axial pull can this spring maintain if the maximum intensity of stress per square inch is not to exceed 20,000 lbs.? (B. of E. App. Mech.)

This problem is easily solved by applying the formula,

$$W=\frac{f_s d^3}{2\cdot55D},$$

where W=the load or axial pull, say, in lbs.;
d=diameter of the coil in inches;
D=mean diameter of the spring in inches;
f_s=shear stress in the material in lbs. per square inch.

Now $d=\frac{3}{8}$ inch$=\cdot375$ inch; D=6 inches, and f_s=20,000 lbs. per square inch.

Substituting these values,

$$W=\frac{20,000\times\cdot375^3}{2\cdot55\times6}=\underline{69\text{ lbs.}}$$

∴ Direct axial pull=69 lbs. Answer.

(2) Suppose that the spring in the previous question were made of square section instead of round, the side of the square being $\frac{3}{8}$ inch. What would then be the axial pull?

For a spring of square section we use the formula,

$$W=\frac{f_s S^3}{2\cdot4D},$$

where S=length of side of square;

the other letters representing the same quantities as before.

Substituting the known values,

$$W=\frac{20{,}000\times\cdot375^3}{2\cdot4\times6}=\underline{73\cdot3 \text{ lbs.}}$$

$\therefore$ Direct axial pull$=\underline{73\cdot3 \text{ lbs.}}$ ANSWER.

The square spring is thus seen to be very little stronger than the round spring, though its sectional area is greater in the proportion of 1 to $\cdot785$, or, roughly, 4 to 3.

(3) A steel helical spring with square coils has a mean radius of $1\frac{1}{4}$ inches, and contains 20 coils. The section of the coils is a $\frac{1}{2}$-inch square. Find how much this spring would extend under a load of 1000 lbs.

To solve this question, use the formula,

$$\delta=\frac{WD^3N}{2{,}135{,}000S^4},$$

where δ=the extension in inches;
W=the load in lbs.;
D=mean diameter of spring in inches;
N=number of coils;
S=length of side of square of the section.

Now W=1000 lbs., $D=2\times1\cdot25=2\cdot5$ inches, N=20, S=$\cdot5$ inch. Substituting these values in the formula,

$$\delta=\frac{1000\times2\cdot5^3\times20}{2{,}135{,}000\times\cdot5^4}=\underline{2\cdot34 \text{ inches.}}$$

$\therefore$ Extension$=\underline{2\cdot34 \text{ inches.}}$ ANSWER.

(4) Design a closely-coiled helical spring to comply with the following conditions:—Spring to be made out of round steel wire, the mean diameter being twelve times the diameter of the wire. Modulus of rigidity of material, 11,000,000 lbs. per square inch. The spring is to stretch 4 inches under a load of 40 lbs., with a shear stress of 20,000 lbs. per square inch.

We may apply the formulæ given in the text to solve this problem.

The formula connecting the stress with the load for a spring of round section is,

$$W=\frac{f_s d^3}{2\cdot55D},$$

where W=the load in lbs.;
f_s=shear stress in lbs. per square inch;
d=diameter of the coils in inches;
D=mean diameter of spring in inches.

Now W=40 lbs., and f_s=20,000 lbs. per square inch. Also, since the mean diameter of the spring is to be twelve times the diameter of the wire, then D=12d.

Substituting these values in the formula,

$$40=\frac{20{,}000\times d^3}{2\cdot55\times12d}.$$

$$\therefore\ 20{,}000d^3=40\times2\cdot55\times12d.$$

$$d^2=\frac{40\times2\cdot55\times12}{20{,}000}=\underline{\cdot0612,}$$

$$\therefore\ d=\sqrt{\cdot0612}=\underline{\cdot25\text{ inch}}\text{ (approx.)}.$$

Hence, the wire will be ·25 inch diameter, and the spring

$$12\times\cdot25=\underline{3\text{ inches}}\text{ (mean diameter)}.$$

The formula connecting the extension of the spring with the load is

$$\delta=\frac{8\mathrm{WD}^3\mathrm{N}}{\mathrm{G}d^4},$$

where δ=the extension in inches;
N=the number of coils;
G=modulus of rigidity.

(*Note.*—The student must in this example avoid using the formula

$$\delta=\frac{\mathrm{WD}^3\mathrm{N}}{1{,}500{,}000d^4},$$

since this assumes a value for the modulus of rigidity of 12,000,000 lbs. per square inch, whereas the steel for the spring under consideration has a modulus of 11,000,000 lbs. per square inch.)

Now δ=4 inches, W=40 lbs., D=3 inches, and d=·25 inch.

By substituting these values, we can find the value of N.

Thus,
$$4=\frac{8\times40\times3^3\times\mathrm{N}}{11{,}000{,}000\times\cdot25^4}$$

$$\mathrm{N}=\frac{4\times11{,}000{,}000\times\cdot25^4}{40\times3^3\times8}=\underline{20}\text{ (nearly)}.$$

$\therefore$ Number of coils=$\underline{20}$.

Having found the number of coils, we can easily find the length of wire required to make the spring.

Thus, $\text{Length} = \pi \times D \times N$ (approx.)

$= \pi \times 3 \times 20 = 188{\cdot}5$ inches

$=$ 15 feet $8\frac{1}{2}$ inches.

$$\left.\begin{array}{r} \text{Mean diameter of spring} = \underline{3 \text{ inches.}} \\ \text{Diameter of coils} = \underline{\tfrac{1}{4} \text{ inch.}} \\ \text{Number of coils} = \underline{20}. \\ \text{Length of wire} = \underline{15 \text{ feet } 8\tfrac{1}{2} \text{ inches.}} \end{array}\right\} \text{ANSWERS}$$

Chapter XIV

STRENGTH OF THICK CYLINDERS

In Chapter V., dealing with the strength of cylindrical vessels exposed to fluid pressure, we investigated the case of a thin vessel such as a boiler shell, and we deduced the important relation,

$$f_t = \frac{pd}{2t},$$

where f_t = the tensile stress in the shell in lbs. per square inch;
p = the internal pressure in lbs. per square inch;
d = diameter of vessel in inches;
t = thickness of shell in inches.

Now, in obtaining this formula we assumed that the stress was uniformly distributed over the longitudinal section of the vessel, and this assumption is quite a reasonable one provided the shell is thin.

When, however, we have to deal with vessels whose thickness is not very small in comparison with their internal diameter—hydraulic cylinders, for example—we are not justified in assuming that the stress is uniformly distributed over the section where we suppose rupture will occur, because the inside portions of the metal have to withstand much greater forces than the outside portions. As a matter of fact, if a cylinder is made sufficiently thick, the outer portions of the metal may play no part whatever in resisting the tendency to bursting.

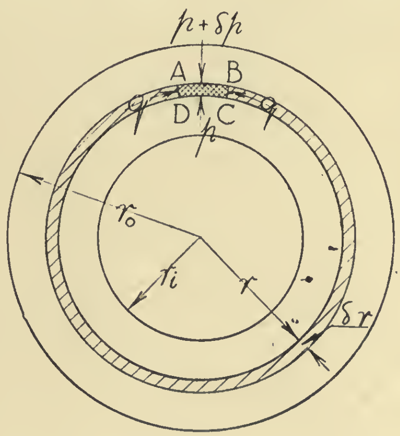

Fig. 75

Thick cylinders must therefore be treated differently from thin cylinders, and we now proceed to obtain a formula which is applicable to such cylinders.

Fig. 75 represents a section through a thick cylinder.

Consider an elementary ring of the cylinder of internal radius r

and thickness δr, the ring being of unit depth or thickness in the direction of the length of the cylinder.

To avoid trouble with positive and negative signs, it is usual in this investigation to suppose that the pressure on the outer surface of this elementary ring is slightly greater than that on the inner surface.

Therefore, let p be the radial compressive stress on the inner surface of the ring, and $p+\delta p$ the stress on the outer surface. Further, let q be the tangential compressive stress in the material; i.e. the crushing stress acting in a direction perpendicular to the radius.

Now consider the equilibrium of one half of the ring.

The force tending to burst the ring is equal to the pressure by the diameter, i.e. to pd (if d is the diameter of the ring), which is of course equal to $p\times 2r$.

Opposed to this there is a slightly greater force of amount $(p+\delta p)\times 2(r+\delta r)$.

The difference between these two forces must be balanced by the forces due to the stress q, viz. $2q\,.\,\delta r$.

Hence, $$(p+\delta p)\times 2(r+\delta r)-2pr=2q\,.\,\delta r,$$

i.e. $$2pr+2r\,.\,\delta p+2p\,.\,\delta r+2\,.\,\delta p\,.\,\delta r-2pr=2q\,.\,\delta r.$$

The quantity $2\delta p\,.\,\delta r$ (a multiplication of infinitesimals) can be ignored, and $2pr$ cancels out.

Hence, $$2p\,.\,\delta r+2r\,.\,\delta p=2q\,.\,\delta r,$$

i.e. $$p\,.\,\delta r+r\,.\,\delta p=q\,.\,\delta r.$$

Dividing through by δr, $$p+r\frac{\delta p}{\delta r}=q.$$

In the limit, $$p+r\,.\,\frac{dp}{dr}=q.$$

Here we have two unknown quantities, and we cannot therefore solve the equation until we obtain another one which we can combine with the first.

Consider any small prism of metal, ABCD in the figure.

On the faces of this prism we have compressive forces acting which tend to elongate the metal in a direction at right angles to the plane of the paper.

These forces are p and q, and they act in two directions at right angles to each other, so that the amount of elongation just referred to is proportional to $p+q$.

We make an assumption here, viz. that a plane cross-section (parallel to the plane of the paper) remains plane after the material has become stressed by the application of the pressure, so that we may take it that the elongation of the prism is independent of r.

If, then, the elongation in question is proportional to $p+q$, we have the further relation,

$$p+q=\text{a constant}=2C,$$

where C represents a constant, the 2 being introduced for convenience of integration. Then

$$q=2C-p.$$

Substitute this value of q in the equation,

$$p+r\,.\,\frac{dp}{dr}=q.$$

Then, $$p+r\,.\,\frac{dp}{dr}=2C-p.$$

$$r\,.\,\frac{dp}{dr}=2C-p-p=2C-2p.$$

$$\therefore\ \frac{dp}{dr}=\frac{2C}{r}-\frac{2p}{r}.$$

Integrating, $$p=C+\frac{K}{r^2},$$

where K is another constant.

Similarly, $$q=C-\frac{K}{r^2}.$$

We have now to find the values of the constants C and K.

Let the internal radius of the cylinder be r_i, and the external radius r_o, and let the internal pressure, reckoned above that of the atmosphere, be p_i, and the external pressure, i.e. at the radius r_o, nil.

Inserting these values for the pressures in the equation,

$$p=C+\frac{K}{r^2},$$

we have, $$p_i=C+\frac{K}{r_i^2},$$

and $$0=C+\frac{K}{r_o^2}.$$

By subtraction, $$p_i=K\left(\frac{1}{r_i^2}-\frac{1}{r_o^2}\right),$$

and $$K=\frac{p_i}{\frac{1}{r_i^2}-\frac{1}{r_o^2}}=p_i\frac{r_i^2r_o^2}{r_o^2-r_i^2}.$$

Substitute this value of K in the equation,

$$O=C+\frac{K}{r_o^2}.$$

Then
$$O=C+\frac{p_i r_i^2 r_o^2}{r_o^2(r_o^2-r_i^2)}=C+\frac{p_i r_i^2}{r_o^2-r_i^2},$$

$$\therefore\ C=-p_i\frac{r_i^2}{r_o^2-r_i^2}.$$

Hence,
$$p=C+\frac{K}{r^2}$$

$$=-p_i\frac{r_i^2}{r_o^2-r_i^2}+\left(\frac{p_i r_i^2 r_o^2}{r_o^2-r_i^2}\right)\frac{1}{r^2}$$

$$=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(\frac{r_o^2}{r^2}-1\right),$$

and
$$q=C-\frac{K}{r^2}$$

$$=-p_i\frac{r_i^2}{r_o^2-r_i^2}-\left(\frac{p_i r_i^2 r_o^2}{r_o^2-r_i^2}\right)\frac{1}{r^2}$$

$$=-\frac{p_i r_i^2 r^2}{(r_o^2-r_i^2)r^2}-\frac{p_i r_i^2 r_o^2}{(r_o^2-r_i^2)r^2}$$

$$=-\frac{(p_i r_i^2 r^2+p_i r_i^2 r_o^2)}{(r_o^2-r_i^2)r^2}$$

$$=-\frac{p_i r_i^2(r^2+r_o^2)}{(r_o^2-r_i^2)r^2}$$

$$=-\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r^2}\right).$$

The minus sign here indicates that the stress is tensile ; so that if we call the tensile stress f, we have finally,

$$f=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r^2}\right).$$

This expression gives the stress at any radius r. The stress will have its greatest value at the inner surface of the cylinder, i.e. when $r=r_i$.

Let f_i be this stress.

Then
$$f_i=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r_i^2}\right)$$

$$=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(\frac{r_i^2+r_o^2}{r_i^2}\right),$$

$$\therefore\ f_i=p_i\frac{r_i^2+r_o^2}{r_o^2-r_i^2}.$$

The stress f_o at the outer surface, where the radius is r_o, will be

$$f_o=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r_o^2}\right)$$

$$=\frac{2p_i r_i^2}{r_o^2-r_i^2}.$$

If f_{max} be the greatest safe tensile stress which may be imposed on the material, then the greatest pressure, p_{max}, to which the vessel may be subjected to with safety is

$$\underline{p_{max}=f_{max}\frac{r_o^2-r_i^2}{r_i^2+r_o^2}.}$$

Thus, $$f_i=p_i\frac{r_i^2+r_o^2}{r_o^2-r_i^2}.$$

Substitute f_{max} and p_{max} respectively for f_i and p_i.

Then $$f_{max}=p_{max}\frac{r_i^2+r_o^2}{r_o^2-r_i^2},$$

and $$\therefore\ \underline{p_{max}=f_{max}\frac{r_o^2-r_i^2}{r_i^2+r_o^2}.}$$

If we take any thick cylinder, and calculate the stress in the material at a number of different radii, we can then plot a curve to show how the stress varies as we go from the smallest to the greatest radius. Such a curve is shown in Fig. 76.

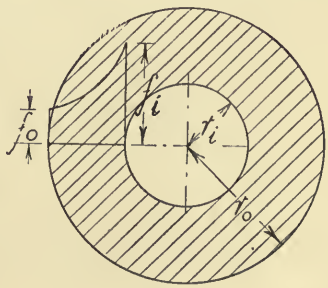

Fig. 76

We see clearly from this that the greatest stress, f_i, occurs at the inner surface, and rapidly becomes less as we approach the external surface. The stress, in fact, varies inversely as the square of the radius, as is evidenced by the formula,

$$f=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r^2}\right).$$

It will be instructive next to consider how the above formula may be applied to the case of a thin cylinder.

Let the radius of the cylinder be R and the thickness of the metal t.

Then r_i in the formula corresponds with R, and r_o with $R+t$.

Now $$f=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r^2}\right),$$

and if we consider the stress, f_i, at the inner surface of the cylinder, i.e. when $r=r_i$, then

$$f_i=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r_i^2}\right)$$

$$=p_i\frac{r_i^2+r_o^2}{r_o^2-r_i^2}.$$

Hence, substituting for r_i the value R, and for r_o the value $R+t$, we have

$$f_i=p_i\frac{R^2+(R+t)^2}{(R+t)^2-R^2}$$

$$=p_i\frac{R^2+R^2+2Rt+t^2}{R^2+2Rt+t^2-R^2}$$

$$=p_i\frac{2R^2+2Rt+t^2}{2Rt+t^2}$$

$$=\frac{p_i\frac{R}{t}\left(1+\frac{t}{R}+\frac{t^2}{2R^2}\right)}{1+\frac{t}{2R}}$$

Now observe that when t is very small in comparison with R, all the quantities $\frac{t}{R}$, $\frac{t}{2R}$ and $\frac{t^2}{2R^2}$ may be neglected, and

$$f_i=p_i\frac{R}{t}.$$

Since $R=\frac{D}{2}$, where D is the diameter of the thin vessel, we have finally,

$$f_i=\frac{p_i D}{2t}.$$

This corresponds with our old formula for the stress in a thin vessel. (See p. 85.)

Worked Examples

(1) A hydraulic cylinder has an internal diameter of 8 inches, and an external diameter of 12 inches. Find the stress in the metal at distances of 4 inches, 5 inches, and 6 inches from the centre of the cylinder, assuming the pressure of the water to be 3000 lbs. per square inch.

The general formula is

$$f=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r^2}\right),$$

where r_i=inside radius of cylinder;
r_o=outside radius of cylinder;
r=any intermediate radius;
p_i=internal pressure;
f=stress at radius r.

Now $r_i=4$ inches, $r_o=6$ inches, and $p_i=3000$ lbs. per square inch.

Then,
$$f \text{ (at 4 inches)} = \frac{3000\times4^2}{6^2-4^2}\left(1+\frac{6^2}{4^2}\right)$$
$$=\underline{7800 \text{ lbs. per square inch.}}$$
$$f \text{ (at 5 ,,)} = \frac{3000\times4^2}{6^2-4^2}\left(1+\frac{6^2}{5^2}\right)$$
$$=\underline{5860 \text{ lbs. per square inch.}}$$
$$f \text{ (at 6 ,,)} = \frac{3000\times4^2}{6^2-4^2}\left(1\times\frac{6^2}{6^2}\right)$$
$$=\underline{4800 \text{ lbs. per square inch.}}$$

This example serves to show how the stress diminishes as we approach the outer surface of the cylinder.

Stress at 4 inches from centre			=7800 lbs. per sq. in.	ANSWERS.
,, 5	,,	,,	=5860 lbs. per sq. in.	
,, 6	,,	,,	=4800 lbs. per sq. in.	

(2) A line of hydraulic pipes, 6 inches diameter, is required to sustain a pressure of one half-ton per square inch. Find the required thickness of the pipes, allowing a safe tensile stress of $2\frac{1}{4}$ tons per square inch.

Here, since we are not concerned with any intermediate radius r, we use the simpler formula,
$$f_i=\frac{p_i(r_i^2+r_o^2)}{r_o^2-r_i^2},$$
where f_i=stress at the inside surface.

Now $p_i=\cdot5$ ton per square inch, $r_i=3$ inches, and $f_i=2\cdot25$ tons per square inch.

We have to find r_o, after which we can state the thickness of metal required.

Substituting the known values,
$$2\cdot25=\frac{\cdot5(3^2+r_o^2)}{r_o^2-3^2},$$

$$2{\cdot}25(r_o^2-3^2)={\cdot}5(3^2+r_o^2).$$

Solving this equation,

$$2{\cdot}25r_o^2-2{\cdot}25\times 9={\cdot}5\times 9+{\cdot}5r_o^2,$$

$$2{\cdot}25r_o^2-{\cdot}5r_o^2=4{\cdot}5+20{\cdot}25,$$

$$1{\cdot}75r_o^2=24{\cdot}75,$$

$$\therefore\ r_o^2=14{\cdot}15,$$

and $r_o=\underline{3{\cdot}76\text{ inches}}$, say, $\underline{3\tfrac{3}{4}\text{ inches.}}$

Thus the thickness of the metal is $r_o-r_i=3\tfrac{3}{4}-3=\tfrac{3}{4}$ inch.

Required thickness of metal $=\underline{\tfrac{3}{4}\text{ inch.}}$ ANSWER.

(3) The internal diameter of a hydraulic cylinder is 8 inches, and the ultimate strength of the material of which it is made is 16,000 lbs. per square inch. What thickness of metal would be required in the sides of such a cylinder if the metal be not stressed beyond one-sixth of its ultimate strength, the water being under a pressure of 2000 lbs. per square inch? Prove the formula which you employ. (Hons. S. and A. Exam.)

The greatest stress occurs at the inside surface, where $r_i=4$ inches, and this stress is not to exceed one-sixth the ultimate strength of the material, viz. $\frac{16000}{6}=2667$ lbs. per square inch,

Again use the formula,

$$f_i=\frac{p_i(r_i^2+r_o^2)}{r_o^2-r_i^2}.$$

Substituting the known values,

$$2667=\frac{2000(4^2+r_o^2)}{r_o^2-4^2},$$

$$2667(r_o^2-4^2)=2000(4^2+r_o^2)$$

$$2667r_o^2-2667\times 16=2000\times 16+2000r_o^2,$$

$$2667r_o^2-2000r_o^2=(2667+2000)16,$$

$$667r_o^2=74672,$$

$$\therefore\ r_o=\sqrt{\tfrac{74672}{667}}=\sqrt{112}=\underline{10\tfrac{1}{2}\text{ inches}}\text{ (approx.)}.$$

$\therefore$ Thickness of metal $=r_o-r_i=10{\cdot}5-4{\cdot}0=\underline{6\tfrac{1}{2}\text{ inches.}}$ ANSWER.

The question also asks for the proof of the formula employed. This is given in the text.

(4) The ratio of the external to the internal diameter of a large gun is 2 to 1. The stresses at radii of 7·5 and 9 inches are

respectively 21·2 and 16·5 tons per square inch, and the maximum pressure is 17·85 tons per square inch. Find the internal and external diameters, and the maximum stress.

The general formula is

$$f=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r^2}\right),$$

where f=stress at radius r;
p_i=pressure at inner surface, i.e. the maximum pressure;
r_o=outside radius;
r_i=inside radius.

In this formula there are two unknowns, viz. r_o and r_i.

We know, however, that $\frac{r_o}{r_i}=2$.

Take first the stress at a radius of $7\frac{1}{2}$ inches. We are told that this is equal to 21·2 tons per square inch.

Hence,
$$21{\cdot}2=\frac{p_i r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{7{\cdot}5^2}\right).$$

Since $\frac{r_o}{r_i}=2$, then $r_o=2r_i$. Therefore substitute $2r_i$ for r_o in the formula.

Then,
$$21{\cdot}2=\frac{p_i r_i^2}{(2r_i)^2-r_i^2}\left\{1+\frac{(2r_i)^2}{7{\cdot}5^2}\right\},$$

$$\therefore\ 21{\cdot}2=\frac{p_i r_i^2}{4r_i^2-r_i^2}\left(1+\frac{4r_i^2}{7{\cdot}5^2}\right),$$

i.e.
$$21{\cdot}2=\frac{p_i}{3}(1+{\cdot}0712r_i^2),$$

$$21{\cdot}2={\cdot}333p_i+{\cdot}0237r_i^2p_i \quad . \quad . \quad (1).$$

This is an equation containing two unknowns, so we require another one.

We are told that the stress at a radius of 9 inches is 16·5 tons per square inch.

Hence,
$$16{\cdot}5=\frac{p_i r_i^2}{(2r_i)^2-r_i^2}\left\{1+\frac{(2r_i)^2}{9^2}\right\}$$

i.e.
$$16{\cdot}5=\frac{p_i}{3}(1+{\cdot}0493r_i^2),$$

$$16{\cdot}5={\cdot}333p_i+{\cdot}0164r_i^2p_i \quad . \quad . \quad (2).$$

Subtracting this equation from (1),

$$4{\cdot}7={\cdot}0073r_i^2p_i.$$

Now $p_i = 17 \cdot 85$ tons,

$$\therefore\ r_i = \sqrt{\frac{4 \cdot 7}{\cdot 0073 \times 17 \cdot 85}}$$

$$= \underline{6 \text{ inches.}}$$

Since $r_o = 2r_i$,

then $r_o = \underline{12 \text{ inches.}}$

$\therefore$ Inside radius = 6 inches,

and Outside radius = 12 inches.

The maximum stress may now be found from the formula,

$$f_i = p_i \frac{r_i^2 + r_o^2}{r_o^2 - r_i^2}.$$

Substituting the known values,

$$f_i = 17 \cdot 85 \left(\frac{6^2 + 12^2}{12^2 - 6^2} \right)$$

$$= \underline{29 \cdot 7 \text{ tons per square inch.}}$$

$$\left.\begin{aligned} \text{Internal diameter} &= \underline{12 \text{ inches.}} \\ \text{External diameter} &= \underline{24 \text{ inches.}} \\ \text{Maximum stress} &= \underline{29 \cdot 7 \text{ tons per square inch.}} \end{aligned}\right\} \text{ANSWERS.}$$

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